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Cybernetic Theory of Cognition and Learning¹

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Introduction

This paper presents a cybernetic theory of conversational interaction and describes one embodiment of that theory CASTE (Course Assembly System and Tutorial Environment). The work which has led to the present development of the theory has been amply represented in the literature; in particular Lewis and Pask [65], Pask, G. "A Cybernetic Experimental Method and Its Underlying Philosophy" *Int. J. Man-Machine Studies* (1971, 279-337) and Pask and Scott [64, 65]. These papers describe some of the work on adaptive teaching and training methods which served as an impetus for the present research.

The theory of conversations and the CASTE system were developed side by side. Each could stand on its own but the theory in effect tells "why" CASTE works; and CASTE shows dramatically "how" the theory may be applied. The remainder of the introduction gives the barest rudiments of the theory, not to explain but merely as an orientation for the reader.

Part 1 describes CASTE and some of the experimental results using the system. Part 2 describes the theory of conversations first by describing its elements and developing a series of ikons that serve as a formal notation and by interpreting the ikons for the experimental and observational paradigms of psychology. Finally a series of Appendices are available for the reader interested in pursuing certain sections of the theory in more depth.

A conversation involves individuals who participate in a process of problem solving, learning and understanding with respect to a set of topics; the domain of the conversation. The basic tenet of the theory is that all psychological observations are observations of conversations, albeit with very restricted domains. Insofar as conversational interactions are observable, they have the form of a stable organization.

In the theory, the stable organization is characterized as a set of procedures (program like entities) that are executed in particular processors such as brains or computing

¹ Presented at the First European Meeting on Cybernetics and Systems Research at a symposium in the Mozart Room of the Palais Palfy organized by Prof. F. de P. Hlanika and Prof. R. Trappl, Vienna, 1972.

machines. Thus a conversation involving 2 human beings, is conceived as a set of procedures executed in parallel in physically distinct processors.

These considerations lead to 2 distinct ways of characterizing individuals.

The first is familiar: "an individual is a human being regarded as a biological unity." This is an *M* Individual or mechanically characterized individual.

The other characterization is in terms of procedures or program like entities, which are *P* Individuals or psychologically characterized individuals, by the following definitions:

a concept is defined as a procedure for reproducing a relation

a memory is defined as a procedure for reproducing a concept

a *P* Individual is defined as a procedure for reproducing a class of memories

A *P* Individual, as a procedure, is run or executed in some *M* Individual but there is not necessarily a one-to-one correspondence between *P* Individuals and *M* Individuals.

Thus a conversation, as a stable organization is a *P* Individual distributively run in 2 (or more) *M* Individuals.

Part 1: CASTE

CASTE (Course Assembly and Tutorial Environment) is a multi-purpose computer based facility which has been used as a course assembly system, a teaching system and as an experimental facility for studying cognition and learning. In the first case the theory specifies explicit principles for the construction and evaluation of course materials and for individualizing instruction. The CASTE teaching mode exemplifies the conditions required for bringing about a tutorial conversation (between student and mechanized teaching system or human teacher) which is guaranteed to provide the student with full understanding of some given academic subject matter. Some results of experiments on human learning using CASTE are presented later in this paper.

The equipment shown in plate 1 is the student's working station and in plate 2 the experimenter's control equipment and viewing window. Most of the regulatory heuristics which govern and monitor learning in CASTE are executed by local equipment but the supervisory routine and some explanation matching sub-routines may either be executed by the experimenter or delegated to a remote computing installation; in the latter case CASTE functions as a very sophisticated CAI system.

CASTE is furnished with two permission-giving (normative) structures that specify the subject matter of an experimental or tutorial session. These are an entailment structure, the large graphic display in plate 1, which stipulates what topic relations may be known or learned; and a set of task structures, one to each topic labeled in the entailment structure which stipulate what may be done, or literally, how each topic relation may be explained. Together, these structures constitute a two level description of the conversational domain, *R*, notably $D^1(R)$ and $D^0(R)$ as discussed more formally in Section Two.

Nodes in the entailment structure represent topic relations and are labeled in one-to-one correspondence (each name is unique to a relation, no name appears more than once). Directed arcs represent the relations, called "Relational operators" that hold

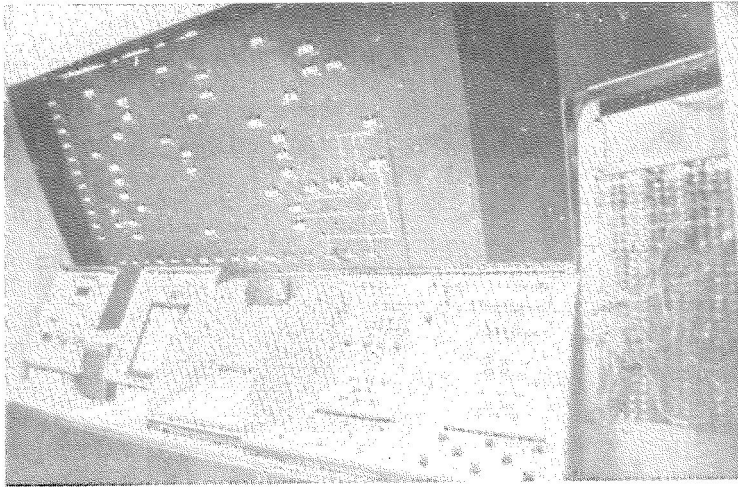


PLATE 1. Student's interface. Entailment structure (on replaceable fascia display) and marking signal lamps.

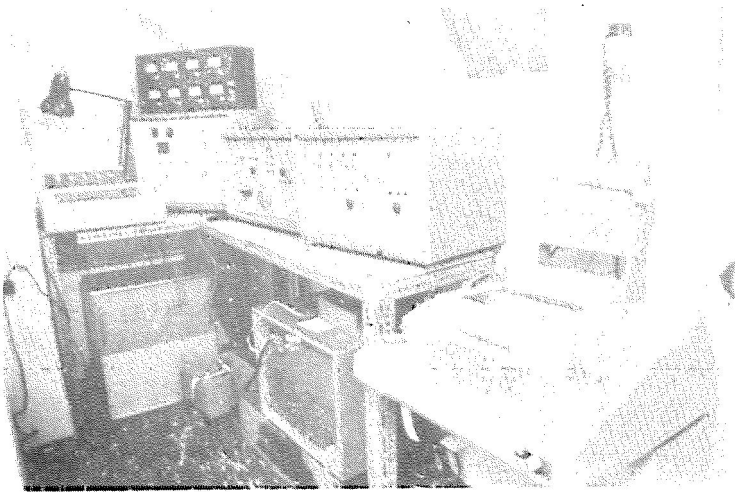


PLATE 2. Experimenter's interface. Local recording and computing equipment with data link.

between them and via which one may be derived from others. Each node is automatically ostendable, either by the CASTE heuristic or (with its consent) by the student and each node may be marked by clusters of illuminated signal lamps (just visible in plate 1) that stand as tokens for the following conditions (a) understood; (b) being aimed for; (c) being worked on (as part of a goal); (d) being pointed at (as a subgoal); (e) being explored. The meanings attached to the token markers are broadly commonsensical and are detailed in table 1. From any node marked explore there is automatic access to files or slides of descriptive data which exhibits the topic in relation to others and from any

TABLE 1. Main CASTE transactions

Code name	Logical function	Method employed	Restriction imposed	Permission given	Mark used
<i>Explore</i>	Ostension and enquiry into description	Dials node index	None	Description of relational network underlying node is available.	Intermittent white signal lamp
<i>Aim for</i>	Student indicates most distant appreciated node as his current learning goal	Dials node index and qualifies by <i>aim for</i> Retained unless deleted	Node must not be understood If <i>aim</i> is marked. Working set must be specified	Allowed to construct working set of nodes entailed by <i>aim for</i>	Intermittent red signal lamp
<i>Work on</i>	Student gives goal description	Dials node index and qualifies by working on	Node must not be understood and must be entailed by <i>aim</i> node. Members of working set may not be deleted, only changed into understood	Allowed to point at subgoal in working on set	Fixed red signal lamp
<i>Point</i>	Student gives subgoal statement	Dials node index and qualifies by point	Node must be working set but concurrent selection permitted	Given access to tutorial data but committed to giving explanation of topic relation of each node	Fixed white signal lamp
<i>Tag aim</i>	Marker used by heuristic in specification of legal working set				Special lamp
<i>Understood</i>	Marker that indicates concept and memory of it	Positioned by heuristic on reception of explanation in the appropriate context	Prohibit <i>aim</i> for or work on. Remove from working set	Provides image of topic relations understood and allows specification of legal working set	Fixed green signal lamp

node marked working on there is automatic access to files of tutorial data that explains or instantiates the topic relation itself.

The subject matter represented in plate 1 is a segment of probability theory. The topic relations are named in table 2 and the entailment structure is readily exhibited in plate 3. Much of the CASTE data referred to in this paper stem from students learning

probability theory but several other subject matters have been employed; notably, logic, strategies of driving, cyclic biochemical processes, mammalian reproduction and a fragment of social science.

Though the entailment structure resembles an hierarchy it is not a partial ordering of topics, like, for example, one of Gagne's [25] knowledge structures. It is, in a very genuine sense, a learnable gestalt. The following sketch of its derivation will exhibit its status.

From subject matter experts or a content analysis of diverse texts we obtain topic names; always these name relations; by using a potentially mechanizable questioning technique we ensure the one-to-one correspondence between names and topics and also exhibit, as exhaustively as possible, how the relations (for example "expected value" or "distribution") are related to one another. The result is a network in which labeled nodes depict relations and directed arcs indicate paths whereby one relation is obtained from others. At this stage each cluster of arcs representing one path is labeled and the labels are represented in unified and formally manipulable terms by considering a finite set of relational operators, one or more of which is used to label each cluster of arcs. Relational

TABLE 2. Key for nodes of entailment structure (Figure 3)

C1: Simple results	I11: Probability Numbers (P. No.) of simple events
C2: Composite results	I12: P. No. of composite events
C3: Exclusive results	I13: P. Nos. of exclusive enteneme
C4: Inclusive results	I14: P. Nos. of inclusive events
C5: Deterministic experiments	I15: Conditional P. Nos. of composite events
C6: Random results	I16: Conditional P. Nos. of composite events
D1: Frequency of simple results	I17: Probability theory
D2: Frequency of composite results	I18: P. No. of complement of composite event
D3: Frequency of exclusive results	J1: Model/Real World distinction
D4: Frequency of inclusive results	J2: Inclusive/Exclusive distinction
D5: Conditional frequency of simple results	J3: Arithmetic operations
D6: Conditional frequency of composite results	J4: Universe
D7: Random experiments	J5: Set and subset
E1: Theory of simple results	J6: Complementation
F1: Probability of simple results	J7: Intersection
F2: Probability of composite results	J8: Union
F3: Probability of exclusive results	K1: Long-run stability
F4: Probability of inclusive results	K2: Counting
F5: Conditional probability of simple results	K3: Order
F6: Conditional probability of composite results	K4: Qualities
F7: Probability theory	K5: "One at once" definition
G1: Event set	K6: Fractional numbers
G2: Composite events	K7: Definition of Probability relation
G3: Exclusive events	K8: Definition of given knowledge
G4: Inclusive events	L1: Relation between logic of structure and arithmetic of P. Nos.
G5: Structural model	L2: Relation between P. Nos. and Conditional P. Nos.
G6: Probability numbers	
G8: Complement of composite event	

A1 to A6 and B1 to B6 are peripheral nodes, concerned with giving access to exemplars. They are not described in this paper.

operators are mathematically well defined (Codd [21]). The set employed has the following member operations, compose, join, cyclic join, identify, isomorph, permute, project, restrict, local complement, union, intersection. By applying members of this set (which is, in fact, redundant) it is possible to generate any relation. These operations are used as metalinguistic labels. It is not assumed that a student has (for example) a join operator in his mental repertoire; only that he can carry out mental operations on U , V , X , Y , equivalent to a (metalinguistic) statement " Y is the join of U and V on the X co-ordinate."

Just as it is common practice to group all mental operations under the rubric "discovery," so relational operators are grouped under the term "entailment." For this purpose each label on a cluster of arcs in the relational network is replaced by a distinguished (terminal) node and all arcs in the cluster are aggregated under one heading. Figures 1 and 2 show the two basic structures that arise. The unlabeled arcs are called entailments; they may be conjunctive (figure 1) and disjunctive (figure 2) and since most of them are cyclic the resulting graph is called an entailment mesh.

The subject matter expert is free to choose any node in the mesh as a head node, that names the subject he is anxious to teach, provided it satisfies the following closure requirement (figure 3). For any topic relations named as a head node, this relation entails each of the relations (other than those distinguished as relational operators) that necessarily (by all of many conjunctive paths) entailed it. If this formal requirement is satisfied, then a very strong but crucial logical/psychological condition is also satisfied;



FIGURE 1. *Left.* Conjunctive substructure in entailment mesh; relational operator at distinguished node, U . *Right.* Denuded mesh meaning, " a entails (is organization of) β and γ ."

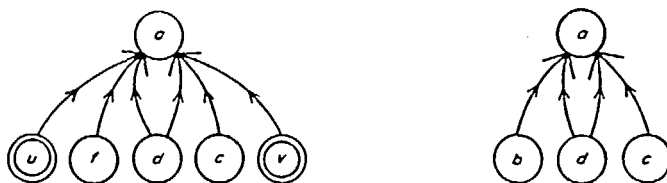


FIGURE 2. *Left.* Disjunctive substructure in entailment mesh; relational operators at distinguished nodes, U , V . *Right.* Denuded mesh meaning " a entails (is organization of) b and d or of d and c or both of these."

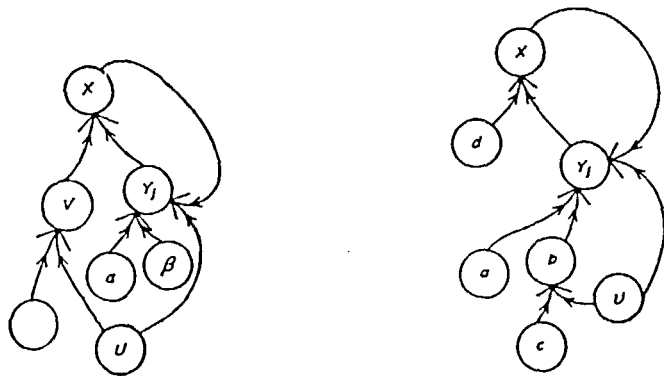


FIGURE 3. Closure condition of nodes labeled X, Y , in entailment mesh. *Left:* U is needed to build concept for Y given concept for X , hence U must be prerequisite for building some concept (for V , in this case) which is required to build concept for X , given concept for Y . But concept for Y may be obtained, also, from concepts for a and β . *Right:* Alternative closure requirement.

the topic explained as the subject matter can be used to explain all of the topic relations that are needed in order to explain it in any permissible manner. The condition is important, for it ensures, that any relation learned by a student, i.e. for which he has built a concept, may be reconstructed or reproduced from relations which he must already have learned.

Given the head node(s) and (optionally) given a number of descriptors, the mesh is converted into an entailment structure such as plate 3. Since closure is guaranteed and taken for granted all reconstruction arcs (other than those depicting internal analogies

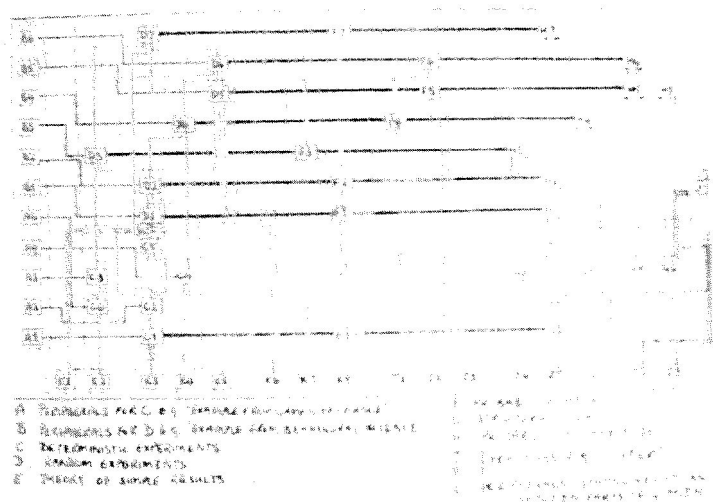


PLATE 3. The "Probability Theory" entailment structure. Nodes are labeled by reference indices in Table 2.

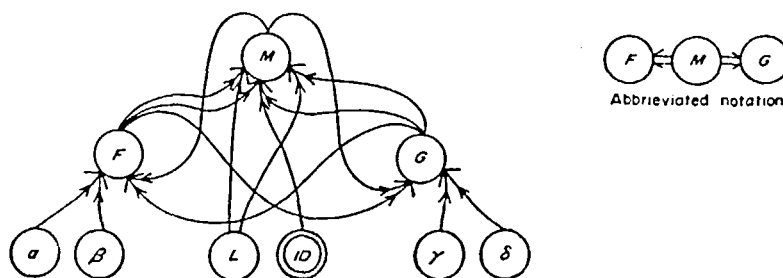


FIGURE 4. Analogy relations to this type remain in the completed entailment structure. M = Analogy in universe, L , between functions or relations F, G , with domain, range, a, β and γ, δ . Given "objects" $a_0, \beta_0, \gamma_0, \delta_0$ in domain, range, of F, G , the structure leads to simple analogical expressions, like " a_0 is to β_0 as γ_0 is to δ_0 ." Distinguished node stands for isomorphism (here, in universe L). Plate 3. Notation on right.

between facets of the subject matter) are deleted and so are arcs going from nodes that entail the head node(s) to nodes that do not. As a result the structure, which is partially ordered except for analogies (figure 4), is disconnected from the mesh; the analogies are represented by a special notation (double lines in plate 3).

The numerical index in plate 3 reflects the partial ordering which represents a whole/part or more/less complex property induced by the choice of head node. A student need not subscribe to this ordering; it is a descriptor on the topic relations, not something inherent in the subject. Other descriptors are tagged by alphabetic indices and name semantic or pragmatic categories; notably (lower right on plate 3) set theoretic models; (upper right) abstract measures called "probability numbers" on pertinent sets; (lower left) real experimental designs giving results; (upper left) the frequency of occurrence of results. Probabilities consist in strict analogies (admitting valid probabilistic inferences only) between set theoretic models and real experimental designs (lower center) and between "Probability Numbers" and frequencies (upper center). In this respect probability theory has a structure which is typical of all the applied sciences we have so far examined.

Free Learning

Obviously CASTE can be used to examine free learning and exploration if the student is allowed unrestricted access to the entailment structure and the data attached to it. This mode of operation is experimentally interesting but has no educational significance, apart from the observation that some students have a built in "teacher" which monitors free learning and renders it effective. The majority of naive students do not have this mental organization and flounder around without any coherent plan or purpose. But experience in the conversational mode (to be described below) does transfer reliably; previously inept students, given a different subject matter in the free learning mode, deal with it as well as the minority group; in other words, training in the conversational mode is one method of "teaching students how to learn."

Conversational Modes

To take part in the conversational modes of CASTE, the student undertakes an experimental or tutorial contract with the experimenter. For his part, he agrees to use the language or transaction system of CASTE, to achieve goals/subgoals that consist either in learning or satisfying the topic relations of the subject matter until the head node is reached; and to regard topics in the entailment structure as relevant (this clause, incidentally, does not preclude the possibility that other topics are relevant in addition). Though we take the student's intention as honest, it is far from obvious that he is able to keep the contract unaided. Hence, for his part, the experimenter agrees to furnish a CASTE heuristic which is guaranteed to so adjust the learning conditions that the student is able to do as he intends.

Two conversational heuristics will be described; one of them instruments the co-operative externalization technique (CET) which has been described in other publications [65]. It satisfies the contractual requirement quite literally and also impels the student to engage in a CASTE language dialogue that externalizes the (normally private) cognitive strategies the student adopts. The other, called the tutorial conversation heuristic, augments the CET heuristic by an uncertainty regulation procedure and guarantees that the student learns as rapidly as he (an individual with certain inbuilt cognitive styles and competences) can, in fact, do so.

We shall describe both heuristics informally, though quite accurately, and ask the reader to accept that they correspond to respectably defined transaction sequences in the stratified command and question language of CASTE which is the object language of the dialogue.

The CET Conversation Heuristic

When a student agrees to learn the subject matter up to its head he is acting under a command to build the necessary concepts and is forced, by the CET heuristic, to answer an explanation demanding questions about how he did so. At any instant, the student can explore (table 1) and obtain data that details the Relational Network, underlying the entailment structure, in the region of the addressed node. Apart from that, he is only required to state the achievement of a concept for some topic relation as his goal. Eventually, often on the basis of the descriptive data, he does so; by marking a node of the entailment structure *Aim*² (table 1). This node may not be already marked understood (essays in this direction are rejected and corrected, in common with other essentially syntactic mistakes) but he may, if he wishes, change his mind and aim for another node; the rule is only that some node is marked aim on each occasion. Given aim, the student is acting under a command to build a concept of the topic relation it

²*Aim* marks a topic relation the student appreciates (as described relative to other topic relations) but which he does not yet understand. In general, several *aim* markers can be placed simultaneously; most students employ one and it greatly simplifies the account if one and only one of them is assumed to exist.

names, and to say how he does so by describing his goal as a set of relations marked working on (table 1). Any topic relations not currently marked understood, can be assigned to the working on set and this set may have one or many members, including the aim node or not. Having placed a node in the working on set, it cannot be removed until it is marked understood (as below). Further, certain restrictions apply to the working on nodes. They must be specifically related, in the entailment structure, to understood nodes. These requirements are calculated automatically. If it happens that a node marked working on is necessarily entailed by any collection of nodes that are currently marked understood, then it is unconditionally accepted. If not, then either the student must demonstrate an ability to explain the relation it names or (failing that) the heuristic recursively applies the aim for procedure, to arbitrary depth, by marking the putative working on node as Tag Aim and demanding the construction of a working on set for Tag Aim. Such nested Tag Aim subsets are processed in order of precedence until a valid working on set is constructed for the original Aim node. The rule, as seen by a student, is that for any Aim node there must be a valid working on set, given which he has asserted his goal.

Given a goal, the student acts under a command to select one or more members of the working on set as subgoals; he does so by marking them *Point*. Subgoals may be selected in any order and several may be dealt with simultaneously. For each subgoal selected the student has access to tutorial data but he is also committed to explaining the relation concerned, i.e. he is acting under a command or explanation-demanding-question to satisfy the relation, not to learn it.

It will be recalled that each topic relation in the entailment structure is associated with a task structure that specifies the (usually many) ways in which this topic relation can be explained. In CASTE, the task structure is a physical analogue for the topic relations on which the student can model (i.e., explain) the topic relation by operations that are electrically sensed and compared with the correct criteria in the task structure. Such devices can be constructed for most subject matters, even rather esoteric ones like historical inference. For probability theory, the equipment is a "Laboratory" called STATLAB (plate 4) on which the student models topic relations by plugging-up operations.

We shall not detail the equipment, which is well documented in the CASTE manual, but append the following remarks.

- (a) Like the entailment structure, STATLAB, is divided into quadrants corresponding to set-theoretic models (lower right) panel; probability number measures and the calculation of resultant measures (upper right panel); real experimental designs which produce results, either from manual operations or from an input tape (lower left panel) and result frequencies and their logical combination (upper left panel). Regional explanations may involve plugging up (modeling) operations on only one panel. Explanations involving probabilistic inference (the upper center and lower center analogy relations in the entailment structure) necessarily span the left- and right-hand panels.

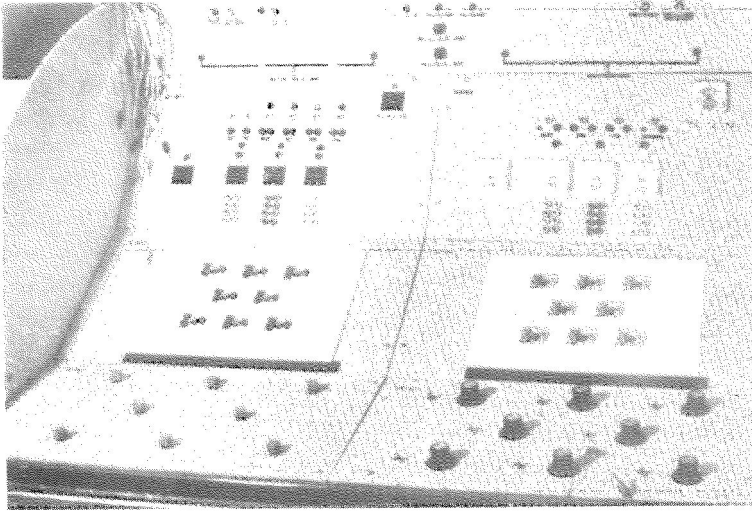


PLATE 4. STATLAB used for the subject matter "Probability Theory" as an analogue of task structure and device for eliciting explanations from student.

- (b) Before using STATLAB as an explanatory tool, the student has invariably encountered demonstrations, instrumented by overlay cards (plate 5), that convey plugging up instructions; he is thus introduced to the techniques of modeling and the conventions of the instrument.

If the student's explanation of a topic relation is correct, i.e. if it matches some explanation in the pertinent task structure, the corresponding node is marked understood, and thereafter it remains marked understood. If his explanation is not correct he receives suitably tailored remedial data.

The rationale behind the marking understood is as follows. In respect of the topic relation (call it R_i) marked point as a subgoal, the student has furnished an explanation of R_i which the CET heuristic accepts as correct because it is compatible with the task structure. Hence, the student has a concept of R_i . But in stating Aim and the goal of his working set, the student has also furnished an explanation of how he constructed R_i and demonstrated an ability to reconstruct it from other topic relations in the entailment structure. Hence, the student has a memory of R_i and, because of that, R_i is marked understood. In the execution of the heuristic, the student may receive more or less co-operative assistance from the data files and the checking procedures but he has certainly re-explained or taught-back R_i . Thus the process we have just described is a teach-back operation, guided by more or less co-operation.

The CET-heuristic, operates in this fashion on all the topic relations addressed by the student and, in that sense, guarantees that they are understood as memories that are a foundation for building up other concepts. Further, insofar as he aims for some topic relation (tutorial contract) and does not aim for a topic relation marked understood (a

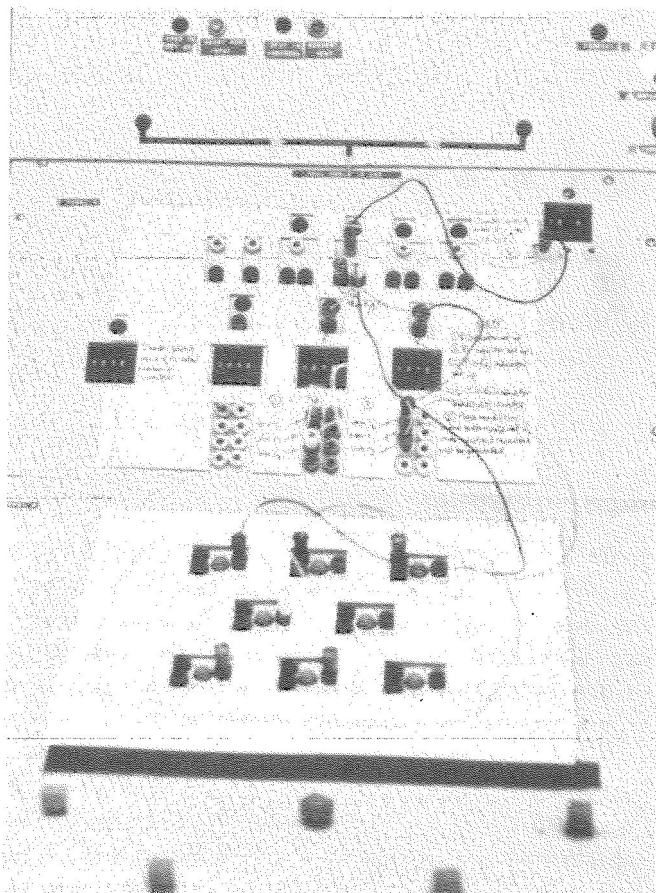


PLATE 5. Overlay card for part of STATLAB, used in demonstrating topic relation.

rule in the heuristic) the student will learn the head topic of the conversational domain. But the necessary entailments of the head node are not pre-specified; nor whether the student learns one or several of them; nor the learning strategy he adopts in order to learn the one or many representations.

On working through this account of the CET heuristic the reader can convince himself that a student who (trivially) knows the subject matter to begin with, exists immediately; on demonstrating an understanding of the first node he aims for, i.e. the head node. For other students, the process is iterated until some entailed set of the head node, and the head node itself, are marked as understood. In order to limit the possible depth of recursion involved in constructing a legal working set of each aim node (and thus to truncate an otherwise unwieldy entailment structure), it is usual to insist that a student can at least understand a bounding set of topic relations (taken as the lower most nodes in the entailment structure) before he starts to learn.

Form of the Learning Process

After learning is complete we have a record of (a) the Aim nodes, (b) the working on sets, (c) the selection of subgoals, (d) the understood nodes, (e) the intervals or efforts (*J*) required for learning; these records are available for each occasion when some change occurs due to a working on node becoming understood. Of these records (a) provides an explanation of how the subject was learned; record (b) and record (c) of how each goal was tackled; record (d) of the students state of knowledge on the occasion in question; and record (e) of the effort he found it necessary to expend. In aggregate, records (a), (b) and (c) delineate the learning strategy the student adopted. It is conveniently represented on a replica of the entailment structure using ▲ to indicate aim. ● to indicate working on (which may overlap aim for) and ■ to represent understood. This notation is employed in plates 6 and 7. A comparable series of images are available to the student in the course of learning when the geometrical symbols are replaced by configurations of illuminated signal lamps (table 1).

Specifically, plate 6 represents, in an objective fashion, the learning strategy called holist. The student who used this strategy was pre-tested (like all students in the study under discussion) and found to be of disposition "holist," as judged by an independent test criterion. (The Gandlemiller test of Pask & Scott, 1972.) Some outstanding features of the holist strategy, observable in plate 6, are the maintenance of a large working set of concurrently accessed topic relations; anticipation, in the choice of an aim node, that is usually well ahead of the working set; the liberal use of analogy relations over and above those essential to learning the subject matter; a tendency to learn a concept in several different ways (exhibited by the selection of several disjointed substructures of the entailment structure).

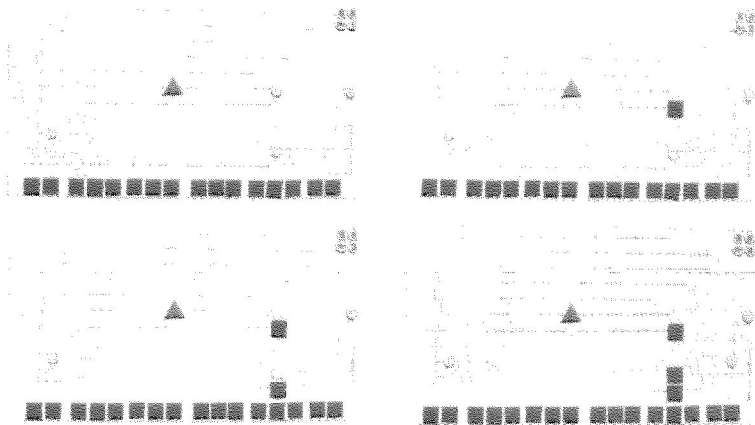


PLATE 6. Holist Learning Strategy for a typical student; one frame to each occasion indexed (1 to 20).

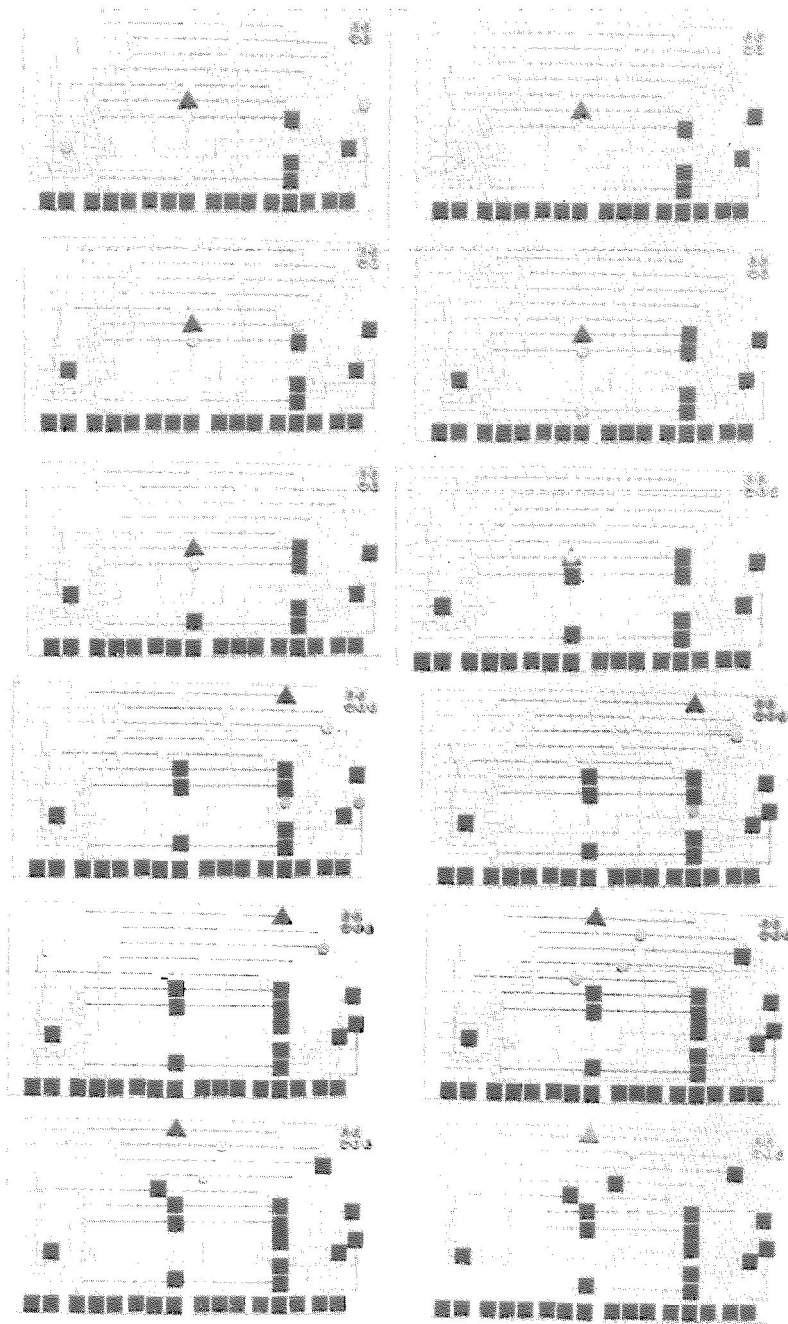


PLATE 6 (continued)

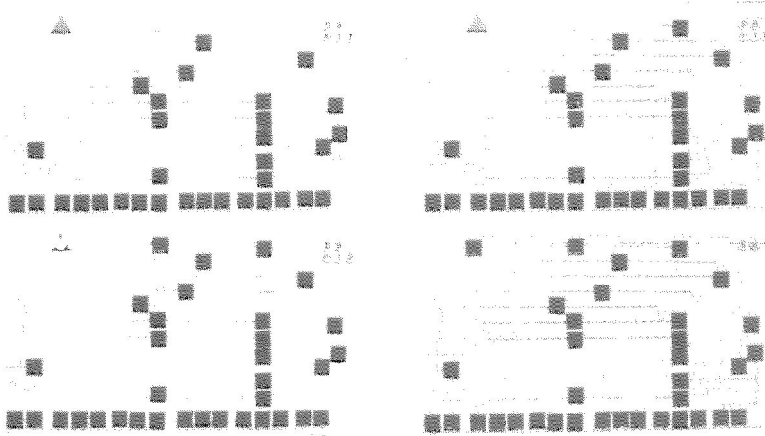
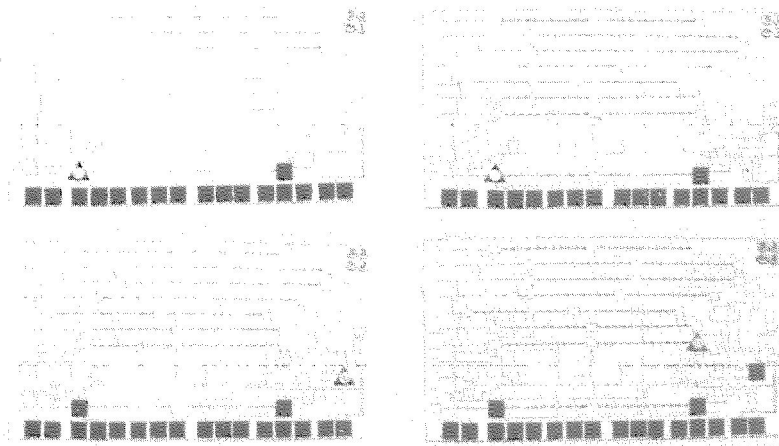


PLATE 6 (continued)

In contrast, plate 7 represents the learning strategy called serialist. The student who used this strategy was found, by pre-testing, to be of “serialist” disposition. The most prominent features of the serialist strategy shown in plate 7 are as follows; the working on set usually contains only one member, topic relations being dealt with one at once; the student generally aims for the node he is working on; the usual construction process is a nesting of topic relations that isolates one of the many entirely conjunctive constituents of the entailment structure.

PLATE 7. Serialist Learning Strategy for a Typical Student; one frame to each occasion indexed by n (1 to 26).

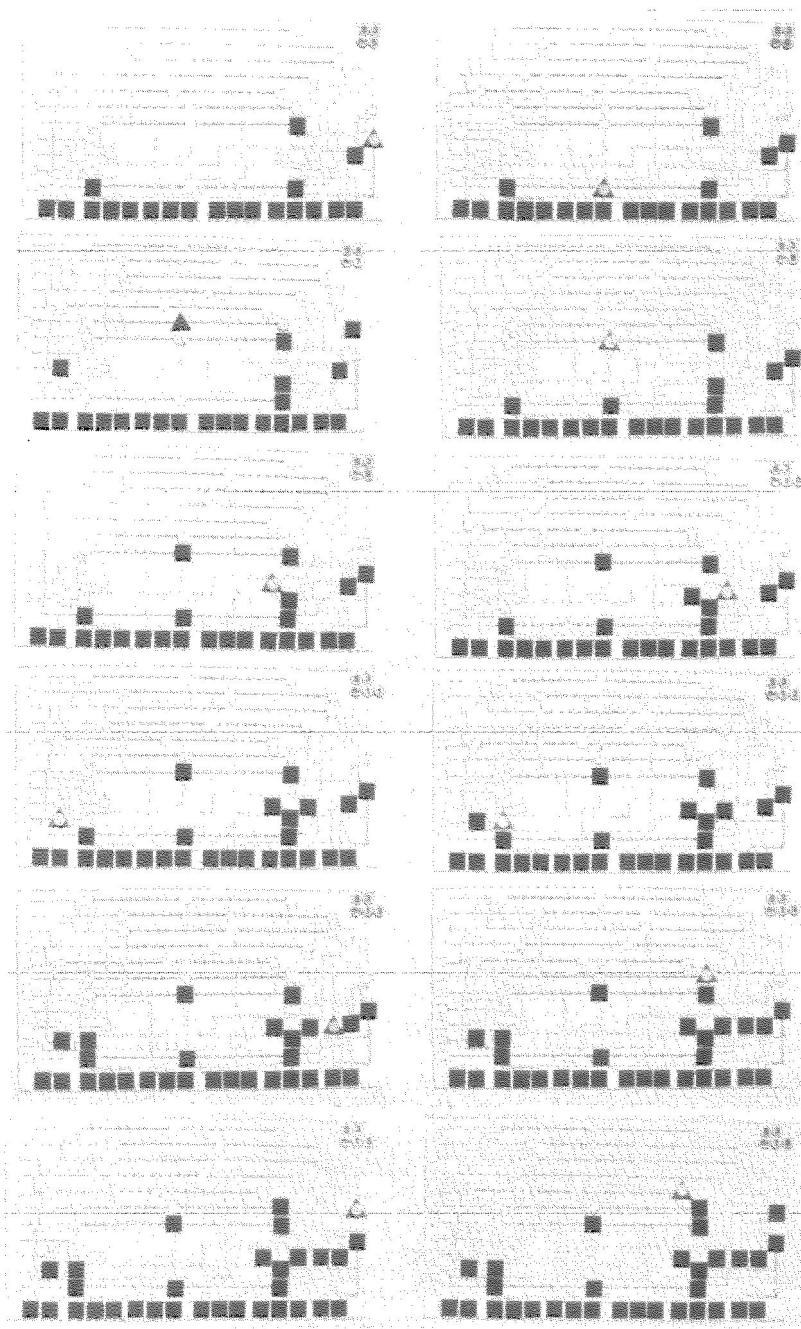


PLATE 7 (continued)

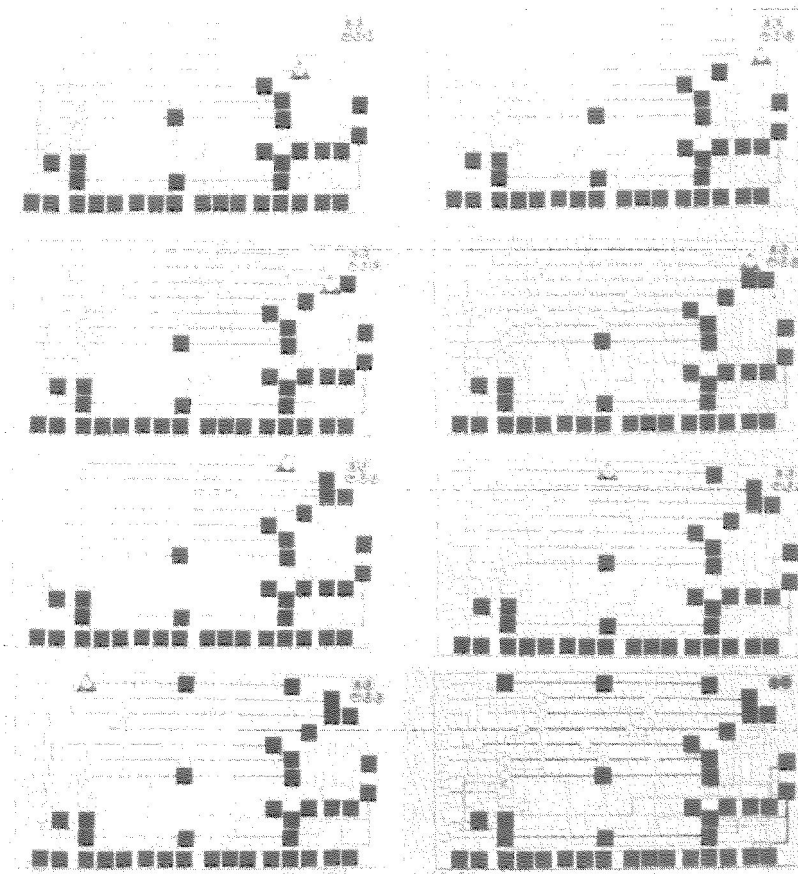


PLATE 7 (continued)

Although the requisite procedures have not yet been described, it is usual to measure indices of problematic uncertainty during the CET conversation even though they are not used directly as control variables. In fact an uncertainty (H) and a correct belief score (θ) are determined for each topic relation worked on and a distinguished uncertainty (H^*) and correct belief (θ^*) are determined for each aim node. These values are tabulated, along with the effort (J) and the number of nodes in the working set for the typical holist (figure 5 in register with plate 6) and the typical serialist (figure 6 in register with plate 7). Notably, the serialist has no uncertainty H^* or correct belief θ^* in respect of an aim node in advance of the topic relation he is working on. Serialist students do, in fact, appreciate topics ahead of those they understand (as we have found by probing in a pre-programmed fashion for correct belief measures). But the region in advance of the current topic relation where correct beliefs are entertained is canalized and often rather

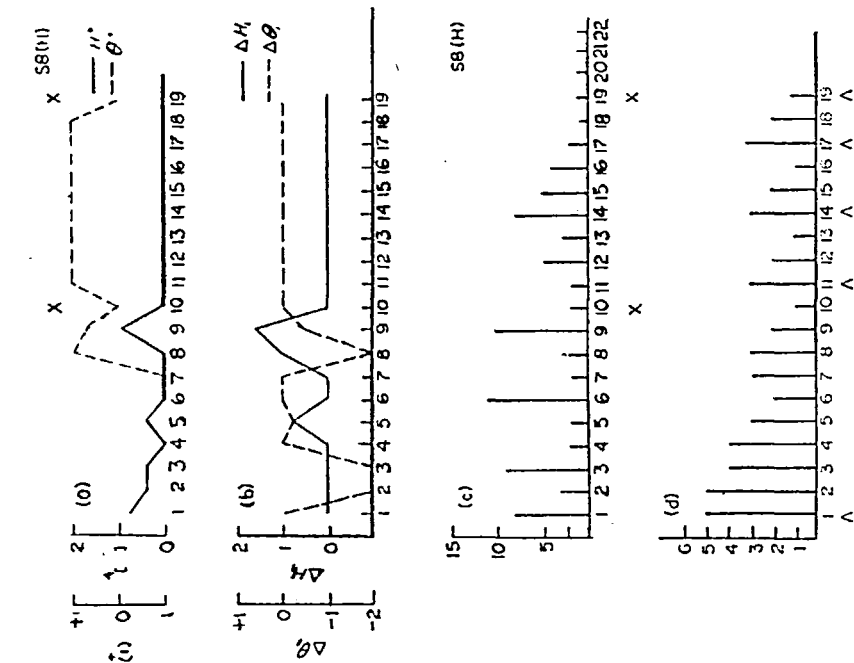


FIGURE 5. Gross data (Plate 6). Horizontal axes show occasion no. Vertical axes as follows:
(a) θ^* and H^*
(b) $\Delta\theta^*$ and ΔH^*
(c) values of θ^*
(d) size of working on set.

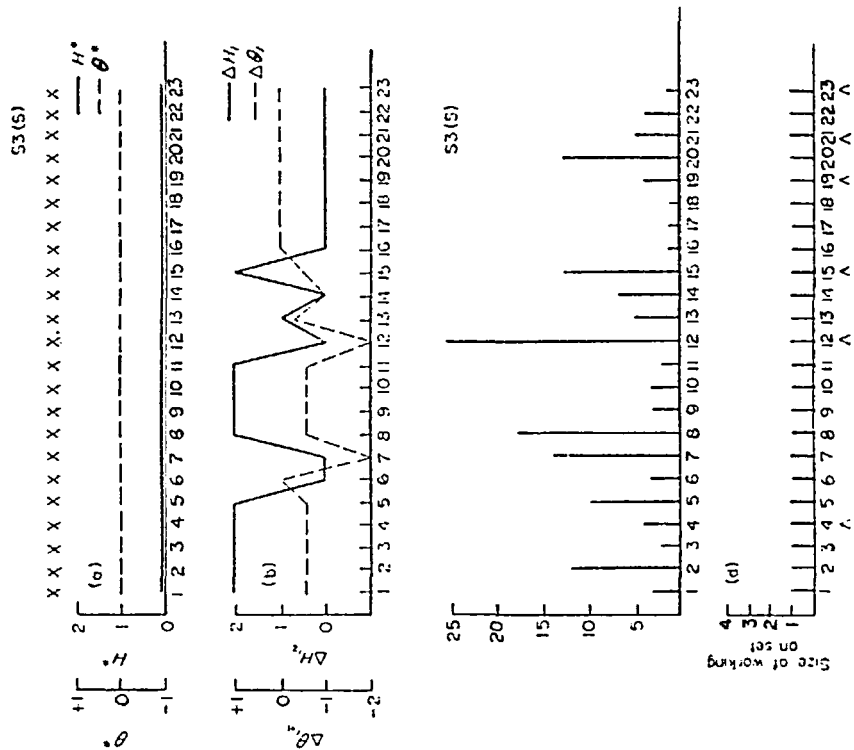


FIGURE 6. Gross data (Plate 7). Horizontal axes show occasion no. Vertical axes as follows:
(a) θ^* and H^*
(b) $\Delta\theta^*$ and ΔH^*
(c) values of θ^*
(d) size of working on set

TABLE 3. Data distinguishing holists and serialists of conversational students

Student No.	Mean H^*	Mean θ^*	Mean size work on set	Gandemuller test score. Max. score = 30
" 1	0.19	0.13	1.0	10
" 3	0	0	1.0	14
" 6	0	0	1.2	6
" 7	0	0.12	1.0	17
"10	0	0	1.3	11
	Mean { 0.04	0.05	1.1	11.6
	S.D. { -0.07	0.05	0.01	3.7
' 2	0.31	0.17	3.0	29
' 4	1.30	0.31	3.4	30
' 5	0.17	0.19	2.8	30
' 8	0.16	0.14	2.6	28
' 9	0.41	0.22	2.4	27
	Mean { 0.47	0.21	2.85	29
	S.D. { 0.42	0.06	0.02	1.2

*Serialists—Gandemuller test: score < 20.

*Holists—Gandemuller test: score > 25.

short; beyond that region, no belief is offered. In contrast, a holist student is quite willing to express beliefs about many topic relations in advance of those that currently concern him and very often those are also correct beliefs. Succinctly, the serialist proceeds from certainty to certainty: the holist also achieves certainty at the points where explanation is required. But these points are embedded in a nexus of dimly perceived but often correctly perceived relations.

As a final point, table 3 summarizes the previously mooted correspondence between pre-test disposition and various indices of strategy type for 10 students. The two groups of 5 serialists and 5 holists were deliberately selected from pre-test data. We do not know to what extent this very close correlation between disposition and type of learning strategy holds for other subject matters.

Part 2: The Theory of Conversations

The basic units of a strict conversation in a language, L , are relations, R_i , with names " i ." Functions are specially restricted relations (one to one or many to one relations) and are treated uniformly. The L description of R_i is $D(R)$ and is a grammar-like, permission-giving, structure which states how R_i may be satisfied. A relation R_i is said to be m -adic or of order m if it unites m distinct properties; a l -adic relation is a property.³

³ Though it would be out of place to argue the point at length in this context this seemingly harmless notion bites at several cherished ground rules of scientific argument and it may be useful to indicate the flavor of the underlying philosophy. For example, one often resorts to the idea of an "unordered set" as somehow elementary. According to this theory (which is in accord with common experience though not with technical parlance) no such thing was ever perceived; in particular, no such thing (though it can be and is conceived) is conceptually primitive or "simple". As a matter of fact, we usually introduce the notion of an unordered set by pointing to a relation, between tea cups on a table, say. Next, we note that apart from the relations that lead us to say they are unitary (because they are all round, made of pottery, so on), they (as units) are arranged in a rectangle, below the chandelier, above the legs of the table. Next, we ask the listeners to conceive all relations like these (an indefinite number) and to discount them; next, to discount the particular relations "above" or "rectangularly arranged;" next, to discount the relations whereby the tea cups are unitary "objects" that may

The notion of a relation (including the limiting instance of a property) is two faceted. From one point of view, germane to physics and mechanics, a relation is a quality possessed by certain configurations of things and not by others or which has distinct (perhaps numerical or quantitative) values with respect to distinct configurations in a fixed universe. In language, this viewpoint leads to an adjectival usage. From another point of view a relation (again including a property) is a procedure which, if executed in a certain universe, operates upon it to bring about or maintain a configuration or which, when presented with a given configuration, may or may not operate (a test, in other words. If the configuration does satisfy the tested condition it is replicated unchanged). The latter point of view is germane to psychology and leads to the linguistic notion of relations as verbs.

To render these ideas more precise consider the language L ; the conversational object language. An m -adic relation R_i is designated adjectivally by an m place L predicate. Pred i ($s_1 \dots s_m$) where $s_1 \dots s_m$ are object variables in L , and Pred i appears in expressions together with the verb "is" of "equality." Under these circumstances the execution of a procedure is tactily referenced: Pred i holds for entities in the universe of interpretation of L because a procedure that maintains R_i is executed iteratively. The adicity of R_i specifies the number of variables or coordinates cited in the same expression to state the condition and the irreducible or minimal adicity of R_i specifies the number that must be cited in order to do so. R_i is designated explicitly as a verb by a procedure Proc i written in L to be executed in an L processor. The execution of a function (one to one or many to one relation) corresponds to the imperative/indicative mood of verb usage; the execution of a relation (other than a function) may (and usually does) involve the conditional/subjunctive mood of verb usage. The irreducible adicity of R_i is the number of loci of control that such an L processor must accommodate at once.

The mathematical and logical representations of a relation are derived as an abstraction from this picture. An intension of R_i is a class of procedures able to compute R_i and an extension of R_i is a standard model for R_i . To derive the extension, each related property or coordinate of R_i is identified with a set of elements $s \in S$, with $j \in J$ an index set. The field of R_i (the union of its domain(s) and codomains(s)) is $\bigcup_{j \in J} S_j$ and R_i is expressed as a subset of the cartesian product $S_1 \times \dots \times S_m$; $j = 1, \dots, m$, so that $\langle s_1, \dots, s_m \rangle \in R_i \subset S_1 \times \dots \times S_m$, the *Universe of modeling*.

Since the general intension of R_i is frequently unbounded, it has little immediate interest. Suppose, however that it is possible to distinguish individuals, A, B (leaving open the question of how A and B are distinguished). In this case A 's (subjective) intension of R_i is a repertoire Proc _{A} i of procedures Proc _{A} k, i , Proc _{A} l, i that bring about or maintain R_i

be patterned in a way that has just been discounted by edict; next, to discount the relations whereby these ghostly remnants of (relational) objects are other than tags (i.e., we have reduced the teacups to an alphabet like A, B, ...; have removed the lexicographic ordering whereby the alphabet is known and all features of the signs A, B, ... except those that keep them distinct). Finally, we rescind the relation "set of alternatives" (over distinct tags). This "unordered set" is such an abstract, purely imaginary, entity that, without detriment, we may impose upon it whatever relation we like to invent. Thus teacups are psychologically real (the theory is not solipsistic) and the objects in an unordered set are psychologically real. But when instancing a set by pointing at real entities we are not, whatever else may be done, pointing at objects.

and B 's (subjective) intension of R_i is a repertoire $\text{Proc}_B i$. Given an L processor and the name i of R_i as a command, a repertoire is executed to generate a model. For example, concentrating upon $\text{Proc}_A i$, its execution generates a model $M_A i$ that is one of A 's extensions of R_i .

The model in question may be abstract or concrete depending upon whether the environment of $\text{Proc}_A i$ is a set of storage locations (holding values of the object variables s_j of L) or a modeling facility such as STATLAB in CASTE in which parts are identified as elements of a universe of modeling selected by $\text{Proc}_A i$. That is, if X stands for the set of parts in a modeling facility, if a^0, b^0 , etc. stand for partially built models then $\text{Proc}_A i(X) = \langle a^0, b^0, \dots, M_A i \rangle$ (provided that it is possible to instantiate R_i in the facility) and $\text{Proc}_A i(M_A i) = M_A i$.

Since $\text{Proc}_A i$ generally consists in several procedures, there are generally different A extensions of R_i namely $M_A k, i$ and $M_A l, i$; synthesized by $\text{Proc}_A k, i$ and $\text{Proc}_A l, i$. In each case the first step delineates a specific universe (X_A which is a subset of X). Similarly, the application of any Proc_B initially establishes the corresponding B universe X_B . So far as A and B are concerned the field of R_i is identified (on the n th occasion) with the universe X_A and X_B . In other words, A and B are free to select their own universes of modeling (perhaps within certain constraints, such as the limited number of event sockets or result sockets on STATLAB). But subject to the overriding restriction that any model built in the facility must belong to a given class M ; for example, the class of finite automata, or the class of mappings or a group or a semigroup. We shall also insist that any model constructed with a special facility such as STATLAB is built up in steps occurring at distinct instants, $t = 0, 1, \dots$. Hence the operation of $\text{Proc}_A i$ (for example) may be represented as the serial application of operators $y \in Y$ (where Y belongs to a mapping from the power set of X onto X ; that is $Y \subset P(X \times X)$). Notice, this seriality condition is not applied to all models; merely to those constructed in a specially reserved modeling facility.

This conception of modeling is at odds with some precepts of cognitive psychology where it is usual for the experimenter to give certain properties to which respondents are supposed to attend; for example, the defining attributes of a universe of Bruner Cards, like color, size and shape. In the theory of strict conversations, individuals are required to select their own universe and place its elements in correspondence with the field of R_i ; though A and B may select any universe that might be employed in a standard experiment; also A may ask B to attend to a specific universe or vice versa. The distinctive difference is that in a standard experiment predication lies outside the proceedings (the experimenter selects and asserts the L predicates and the properties they denote); in the conversational situation predication figures as a crucial part of the dialogue and is due to the participants.

Although the act of predication is not discarded as irrelevant by standard psychology it usually either remains unmentioned or is assigned to a special compartment of attention;⁴ studies in outstanding exception is the brand of psychology [40; 7] concerned with

⁴ Rightly, in a sense, since predication is attention. But the majority of attention studies beg the real question by assuming that man is furnished with a great variety of perceptual filters from which selections are made when someone "directs his attention." The trouble, here, is that a brain is implicitly tagged as a special purpose computer with prescribed

personal constructs which are produced by predication and are frequently elicited by the repertory grid technique.

Another departure from the standard approach is marked by the fact that M_{Ai} and M_{Bi} may either be static or dynamic models. As in the case of predication, there is no real dispute or disparity between standard psychology and the present theory since it is fairly uncontentious to say that any relation must be maintained if it is to persist. Only sometimes it is legitimate not to mention the mechanism that maintains it; when, for example, there is a principle of invariance or when the elements of a universe are inherently stable (a fact mathematically imaged by attaching an identity operation to each one). So, for example, the relations "North of" and "rectangular" are legitimately given static models insofar as they are instantiated in terms of objects that remain fixed in space once they are positioned. Other relations are explicitly preserved by executing the same operation; for instance, the relation between room temperature and a thermostat setting is maintained by a central heating system and, in general, such relations are stabilized by a control mechanism that expresses the relation. Here, a model for the relation (one of its extensions) is dynamic and is executed in time so that R_i (in this case quite explicitly) holds over a period of time. Whereas standard psychology usually takes the legitimacy of a static model for granted, one cannot afford to do so when discussing strict conversations and the explanatory facet of dialogue.

The distinction between static and dynamic models is artificial since all models should, strictly, be dynamic. Static modeling is a convenient shorthand form of representation and no more than that. Provided this fact is recognized the use of static models where they are legitimate avoids a great deal of cumbersome symbolism.

The following examples use the relation " $\oplus$ " or "add modulo 4" as a vehicle for illustrating both the static/dynamic distinction of immediate concern and the general notion of a universe of modeling. One model of " $\oplus$ " (A 's perhaps) is defined over a limited universe of integers. Its extension, table 4, is a list of all triples $\langle s_a, s_b, s_c \rangle \in S_a \times S_b \times S_c$ of all integers in S_r ; $r = a, b, c$; $S_r = \{1, 2, 3\}$ for which the relation holds.

This figure might be drawn in terms of numerals, simply distinct from one another, and members of a relation on $P(\{1, 2, 3\})$ called "set of alternatives." As it is, however, the triples have been structured by assigning the term sum to the last coordinate (or column) S_c and the term summands to the first pair S_a, S_b . Moreover the values of each coordinate have been structured under a successor operation and it is thus possible to assert that $3 > 2 > 1$. The model is readily generalized so that it represents "+" of any modulus over an arbitrary number system (integers, rationals, reals) which might be B 's universe. But the weak version is illuminating insofar as it typifies the kind of relation

though redundant input, rather than a general purpose computer for which input and program variables must be declared. Hence, the genesis of relevance is consigned, in turn, to biological or developmental studies. Computer scientists generally view the matter differently. Since they deal with general purpose (programmable) machines they appreciate that variables must be declared and that in order to declare them there must be (beforehand) a relation in mind which is instantiated by identifying its field with the universe of these variables. But, of course, it is the programmer who declares the variables (in much the same way that the experimenter declares the attributes in standard cognitive psychology). Our development places the act of predication inside the conversation rather than outside it.

TABLE 4. Addition of remainders module 4 symbolized ($\oplus$) and the 3 tuples that satisfy this relation. Notice that the coordinates of the relation may be more or less specialized; that is, any coordinate may have a relation (such as successor of) assigned to it and as a result, it may be structured.

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Addition Table For $\oplus$

Set of 3 tuples satisfying the relation $\oplus$, a triadic relation

Summands		Sum
S_A	S_B	S_C
0	0	0
0	1	1
1	0	1
0	2	2
2	0	2
0	3	3
3	0	3
1	1	2
1	2	3
2	1	3
1	3	0
3	1	0
2	1	3
2	2	0
2	3	1
3	2	1
3	3	2

Universe of $\oplus$ is $S_A \times S_B \times S_C$ which is included in the universe of additions like $\oplus$, namely $PS(\{0,1,2,3\})$.

that holds over nonmathematical fields. For example, kings of a country are distinct and chronologically ordered and related to other monarchs, dynasties and so on.

A dynamic model for the relation " $\oplus$ " is an adding machine. This, of course, is a process.

Numerical Input $\rightarrow$ Numerical Output

$\oplus$

The process may be represented, canonically, as a finite automaton (clearly there are many addition methods and thus finite automata that add, but all of these specific models are extensionally equivalent, since they realize $R_i = \oplus$). The model is dynamic insofar as given any pair of inputs at instant τ it outputs the sum at instant $\tau + 1$ thus maintaining stable the condition that R_i holds over any interval $\tau, \tau + 1$ where $\tau \in T$ and $\tau + 1 \in T$. The unspecific extension of $R_i = +$ (Modulo 4) is obtained by structuring the coordinates so that the first pair is labeled τ and the last column entry is labeled $\tau + 1$. Notice that addition is either regarded as immaterialized or some such trick as τ

labeling is played in respect of elements in the universe. For instance, the trick played in set theory is to regard elements as integral (the identity axiom) and relations as temporal. An index such as $\tau \in T$ has the effect of materializing addition in a less abstract but still very restricted fashion (the assumption that values of τ are strictly ordered asserts, in effect, that the universe is a finite state machine).

A specific machine (a finite automaton) can also be represented in extension, for example by a set of quadruples.

Input State at τ , (the first pair in table 4)	Internal State, at τ	Internal State, at $\tau + 1$	Output State at $\tau + 1$ (the last entry in table 1 or its generalization.
---	------------------------------	----------------------------------	---

or for terminology $\langle u_\tau, Z_\tau, Z_{\tau+1}, v_{\tau+1} \rangle$ ($u \in U$ input states; $z \in Z$ internal states; $v \in V$ output states; $\tau \in T$ trial order). This account is clearly much more concise than a generalized function table drawn (impracticably) for "all integers" or "all real numbers." However, it is also, necessarily, more specific. The modeling facility must be meaningfully structured to represent states each of which (since one and only one state occurs at once) is a member of a set of alternates (itself, recall, a relation). Concisely, any model of this type belongs to a certain class, here the class of finite state machines. A specific state $x \in X$ (or $x \in X_A$ or $x \in X_B$) is a quadruple.

$$x = \langle u_\tau, Z_\tau, Z_{\tau+1}, v_{\tau+1} \rangle$$

a specific machine, say the l th machine, is a set of quadruples

$$X_l = \{x\}_l = \{\langle u_\tau, Z_\tau, Z_{\tau+1}, v_{\tau+1} \rangle\}_l$$

such that any $x \in \{x\}_l$ is a member of a mapping pair

$$F_l; U \times Z \times T \rightarrow V \text{ (Next output state function)}$$

$$G_l; U \times Z \times T \rightarrow Z \text{ (Next internal state function)}$$

where $F_l \subset X \times T$ and $G_l \subset X \times T$ for any value of l . If the class M which restricts the models that may be built in a certain facility is the class of finite state machines then this is a detailed description of the constraint introduced previously, that any model must be a member of the class M . Notably it is quite legitimate, in a modeling facility, to reduce the specificity either by projecting a relation onto one or more of its coordinates or by assigning a constant value to any one coordinate. For example, if $z = z_0$ for all $z \in Z$ then M accommodates dynamic mappings (in Ashby's sense "Black Box" models); if $\tau = \tau_0$ for all $\tau \in T$, also, then M accommodates all mappings such as a temporal function specifications.

The symbol τ is deliberately reserved for an index over steps or trials in the execution of a dynamic model in contrast to the index t over the instants at which $\text{Proc}_A i$ or $\text{Proc}_B i$ is executed. Thus τ represents the output of a facility clock, like the input tape oscillator in STATLAB or the execution act of pressing one result button. In contrast t represents the output of a clock that progresses by one unit for any stage in modeling.

We generally insist (though, as later, the restriction is not essential) that the τ clock is zeroed ($\tau = 0$) if t changes in value. In other words, if a dynamic model is executed ($\tau > 0$) then model building ceases and if modeling operations are in progress then the trials clock is stopped and zeroed. These constraints secure a structure/function/distinction and preclude amongst other things, the possibility of models that act upon themselves. If these constraints are imposed a modeling operation can be expressed by a series. The l th model $M_{A,l,i}$ built by A at instant t during the n th occasion is

$$M_{A,l,i} = \{x\}_l = \{x\}_t = Y_t(Y_{t-1}(\dots Y_1(Y_0(X)) \dots))$$

obviously (as in CASTE) the model $\{x\}_t$ may be executed ($\tau > 0$) and, if necessary, modified to produce further models $\{x\}_{t+r}$, $r > 0$, before a model is finalized.

Two important types of clocking constraint are exhibited in figure 7 and figure 8. As before, t is the index of a modeling (or operating) clock and τ the index of a model execution clock but there may be more than one of each.

In figure 7, which preserves the structure/function distinction, there is one t clock and one τ clock; further these are stopped and started as described above; τ being zeroed if a fresh model is made (a segment of t values). Consequently, all t and τ segments not only belong to one and only one occasion, n (so that they should properly be subscripted by n), but also they are linearly ordered in n . An external observer can locate the τ th

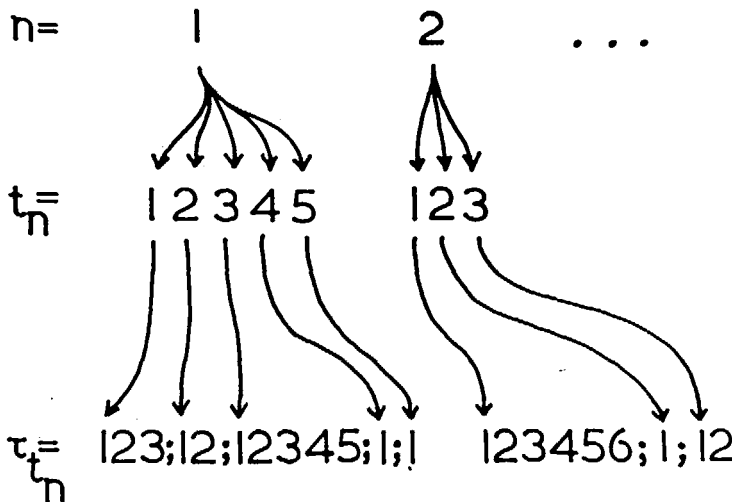


FIGURE 7. Serial model (program) building in serial facility (processor).

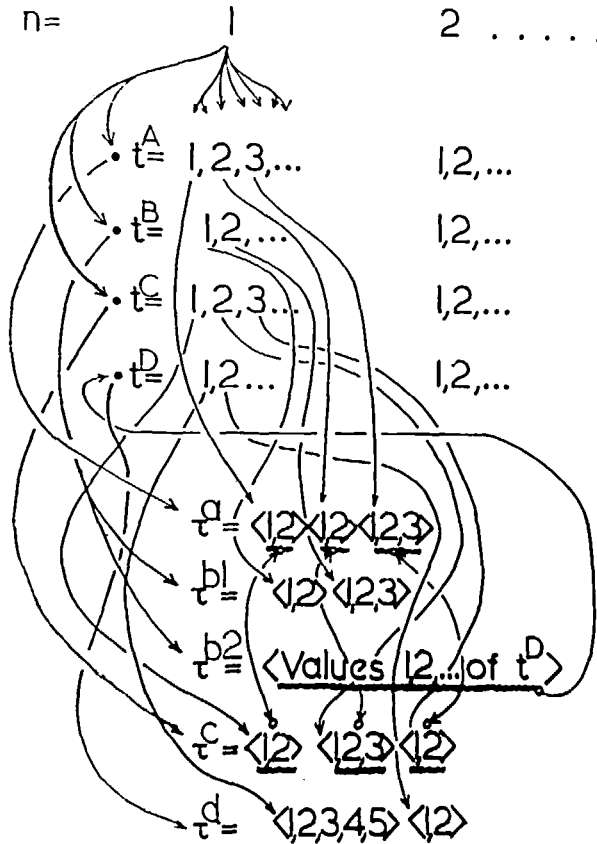


FIGURE 8. Concurrent organization.

execution step in the t th model of occasion n , as suggested by the notation $\tau_{t,n}$ (for execution steps) and t_n (for modeling operations).

In figure 8 illustrating the opposite extreme, the maximum tolerable liberality, there are several clocks, $t^A, t^B, \dots$ so that models can be built concurrently and (if desired) independently since the t clocks are specified as asynchronous (being moved on by modeling operations) unless they become synchronized by virtue of the model maker.

Similarly there are several (unless otherwise prescribed) asynchronous model execution clocks labeled $\tau^a, \tau^b, \dots$ so that the models made under $t^A, t^B, \dots$ can be executed in parallel (some t may have several τ , for example, $t^B \rightarrow \tau^{b1}; \tau^{b2}$) independently of the model maker. The only caveat is that a change of model, that changes t^A , also zeroes τ^a and that a change of t^B zeroes whatever is attached to t^B (in this case τ^b only, since τ^{b2} is also in receipt of an input from elsewhere).

Two kinds of interaction can arise in such a system (1) the operation of τ^a and of τ^c is synchronized because of communication between separately constructed models equipped with initially unsynchronized execution clocks. Later, we develop the fundamental point that information transfer, in an unusual but very relevant sense of that

phrase, is uniquely correlated with synchronizing (communicative) events, that are responsible for coalescing otherwise asynchronous systems. (2) As a rule, t clocks are regulated by the model maker (a participant). Since there may be several τ clocks to one t clock, an important facet of modeling is to recruit as many execution clocks as are required. The convention that t clock operation recruits or initiates execution (τ) clocks is retained but a special case (of appreciable significance) is shown by the illustrated connection from τ^{b2} (and model b) to the t clock, t^D . In other words, using the intermediary of a t clock, a model can make other models (t is usually advanced by modeling operations performed by a participant; here t^D is advanced by the modeling operations of model $b2$). Further execution clocks may thus be recruited. Although the point is not pursued in depth, it should be noted that the execution of a function/relation is tantamount to verb usage. Relations (other than functions) can be executed in the subjunctive mood as well as the imperative/indicative mood. To recruit execution clocks is (roughly) the meaning attached to subjunctive execution.

In figure 8 all segments of the model making carried out and all segments of model execution belong to one and only one occasion n , but they are not linearly ordered in n , nor (since in condition (2) we are tacitly assuming that τ^{b2} is a clock made by modeling and thereafter allowed to run on its own) is there an obvious "ordering" of any kind. However, all t clocks are zeroed when the occasion changes and thus all τ clocks are zeroed at the moment when understanding is detected. The important point is that understanding might occur as a result of concurrent (and potentially communicative) activity in the modeling facility.

We do not, as previously emphasized, admit such liberally conceived modeling facilities. In fact, we remain with modeling facilities clocked like figure 7 because, for example, the strict conotation of state loses its meaning under the conditions of figure 8.

But so far as conversation is concerned, there is no objection to many clocked facilities and they are one stage removed from the many clocked L processors discussed later. These differ from figure 8 in only one respect; there is no requirement that an occasion shall have a beginning and an end; or that, clock-wise, there shall be an occasion.

An Overview of the Language, L

L , the stratified conversational object language is a command and question language. It has developed as an extension of Rescher's [70] theory and includes various types of questions (for example, "how," "why," "what," "which," "whether," question of a form consonant with Harrah's [31] model) and certain combinations of commands and their execution or questions and replies to them that constitute explanations either (a) of how to compute the relations under which the commands are issued and the questions are posed or (b) explanations of how to construct the relations from others. In order to disambiguate, the language is stratified (as suggested by the distinction between explanations of type (a) and (b) into higher and lower level statements).

In CASTE, as mentioned previously these levels are embodied as the Entailment Structure (what may be known) and the Task Structure (what may be done).

The reader may wish, at this point, to look at Appendix B, a discussion of Command and Question Logic.

It is assumed that any individual in conversation has a matching operation which is continually available as a means of determining identity from this individual's point of view (formally, the matching operation is one of the primitive procedures, *Prim*, to be introduced as part of all cognitive repertoires). Any model either is or is not matched to some other model; in particular, it may or may not represent the "same" relation R_i . Generally, there are two simple interpretations for the "same," depending upon how and when the matching operation is applied; namely, an identity of extensional equivalence and an identity of agreement. In a tutorial conversation, where polarity is imposed by the distinction between "teacher" and "student," there is also a real issue of whether or not a model is correct and (if it is correct) of whether or not it is also complete in the sense that it will bring about or satisfy R_i for all elements in a universe identified with elements in the field of R_i .

The demand for building a model is issued by a command; in essence, by an L programming edict. If the command is general, "make any model for R_i ," it is dubbed a Base command. If the command is specialized (either "make a particular kind of model for R_i " or "make a model that does not contravene a class of boundary conditions") then it is qualified. In all cases the command is addressed (" A ! bring about R_i ") to some individual (in this example, to A).

Suppose that A receives the base command " A bring about R_i " under conditions in which it is interpreted as " A make a model for R_i ." Suppose, also, that A has a repertoire, $\text{Proc}_A i$ of procedures, for obeying this command; for example, $\text{Proc}_A k, i$ and $\text{Proc}_A l, i$ which are executed to produce two distinct models in the same universe of modeling (X_A) constrained to be members of the same class (M) and to be built in the same facility; $M_A k, i$ and $M_A l, i$. Even though the procedures $\text{Proc}_A k, i$ and $\text{Proc}_A l, i$ are by prior definition, distinct (they are different subjective intensions of R_i) it is still possible that $M_A k, i$ and $M_A l, i$ are identical by matching, that is, each one can be executed in the facility (M) and each element of $M_A k, i$ corresponds to one and only one element of $M_A l, i$. If these conditions are satisfied $M_A k, i = M_A l, i$ and both $\text{Proc}_A k, i$ and $\text{Proc}_A l, i$ are said to be extensionally equivalent (which justifies the initial assumption that they are both members of $\text{Proc}_A i$). Further, a description of how to build the model $M_A i$ is also (in this case) a description of how A would bring about R_i and is derivable from a sufficiently broad account of how to bring about R_i . Assuming that the description $D(R_i)$ has this much generality for each R_i in the conversational domain R , derivability is signified by $D(R_i) \Rightarrow M_A i$.

In contrast, suppose that A and B are both commanded to bring about R_i in a facility limited by accommodating only class M models: that they are both able to do so, and that they build a pair of distinct models $M_A i$ and $M_B i$. Quite possibly A and B have different universes of modeling (X_A and X_B ; it is certainly true that X_A and X_B can be and are likely to be different). Consequently, it is improper to equate $M_A i$ and $M_B i$ as they may be distinct entities (they might even be built in different facilities). However, if both of them are evoked by a command to bring about the same relation R_i , then A has identified elements in his modeling universe (X_A) with the field of the relation R_i and B

has identified elements in his (possibly different) modeling universe (X_B) with the field of R_i . Under these circumstances a one to one correspondence established by matching is an isomorphism signified by $M_A i \Leftrightarrow M_B i \subset M$ and if this condition is satisfied then A and B are said to be in exact agreement about their models. That is, A and B conceive the extension of R_i as the same relation even though they exhibit it in different universes of modeling (as for example, the reader and I will have a common extensional meaning for words like “circular” and “great uncle” though we are likely to instance them quite distinctly). If A and B are in exact agreement, in respect of R_i then there is at least one procedure in common that yields the same model so that $D(R_i) \Rightarrow M_A i$ and $D(R_i) \Rightarrow M_B i$. In this sense, “ A ’s intension of R_i ” $\Leftrightarrow$ “ B ’s intension of R_i ” for some Proc in $\text{Proc}_A i$ and some Proc in $\text{Proc}_B i$.

The strict requirement of exact agreement can be waived, without detriment, insofar as the isomorphism “ $\Leftrightarrow$ ” is replaced by an R_i preserving homomorphism and, if so, A and B are merely said to “agree with” each other about relation R_i . For exposition, however, it is convenient to restrict attention to isomorphisms.

Conversely, a rather deeper meaning is attached to agreement if the dialogue is engendered by a qualified command requiring that R_i is brought about by a particular class of procedure. If so, agreement involves intensional or programmatic equivalence. For example, one very strict criterion for agreement is to match the productions of A and B ; namely (using superscripts to label the participants) to match each stage in (or each instant t in) a pair of modeling operations; that is to match A ’s operation

$$\text{Proc}_A i(X) \Rightarrow \langle a_A^0, b_A^0, \dots, M_A i \rangle \text{ or (by seriality)} \quad (\text{A-1})$$

$$M_A i = y_{t_A}^A (y_{t_A-1}^A (\dots (y_1^A (y_0^A (X)) \dots)))$$

with B ’s operation

$$\text{Proc}_B i(X) \Rightarrow \langle a_B^0, b_B^0, \dots, M_B i \rangle \text{ or (by seriality)} \quad (\text{B-1})$$

$$M_B i = y_{t_B}^B (y_{t_B-1}^B (\dots (y_1^B (y_0^B (X)) \dots)))$$

matching is easy if and only if $t_A = t_B$. Using rather more sophisticated matching criteria, agreement/disagreement can be established, regardless of production length, provided that all productions are serial, or even provided they have a unique “beginning” and “ending.” But knowledge of program equivalence is in flux at the moment and criteria of this kind are context specific or, if not, they are unduly restrictive.

The statement form “ $M_A i \Leftrightarrow M_B i$ ” is an “ L metaphor” which designates a material analogy and its context is expressible in the usual manner. For any pair of elements α, β , in the domain and codomain (field) of A ’s universe of modeling and any pair of elements γ, δ , in the domain and codomain of B ’s universe of modeling it is asserted that “ α is to β as γ is to δ ,” in which one “is to” stands for R_i in A ’s universe and the other for R_i in B ’s universe and “as” stands for an analogical universe in which A ’s predication is

meaningfully juxtaposed with B 's predication as a matter of agreement due to the existence of a conversation involving both A and B .⁵

In a strict conversation basic statements are " L metaphors" which play much the same part as "veridically true propositions" in a standard language. Only, of course, L metaphors denote material analogies and propositions denote "facts" or "set membership" which are simple relations.

It is important to realize that the notion of correctness in a tutorial conversation is a special case of agreement rather than veridicality. One participant in a conversation in the role of teacher (conventionally B) is deemed an authority insofar as B is in possession of a canonical theory; true or false, it does not matter. This dictum is reified (in CASTE, for example) by writing the theory as the conversational domain, R , and noting that its L description $D(R)$ is, amongst other things, an account of how each R_i in R may be brought about. It is next pronounced that for each R_i in R the teacher's repertoire Proc_{Bi} is the execution of description $D(R_i)$ in some L processor. A normally permission giving structure is thus taken in an imperative form. If A and B are both ordered by a base command to bring about R_i then the agreement $M_{Ai} \leftrightarrow M_{Bi}$ implies that A has made a correct model for R_i in the sense that $\text{Proc}_{Bi}(X) \equiv \text{Execution } D(R_i)$ to produce a class of models $\{M_{Bi}\}$ such that some $M_{Bi} \leftrightarrow M_{Ai}$.

As before, qualified commands are more complicated. A teacher who issues a qualified command insists upon a measure of intensional agreement between an A modeling operation and some production obtained by executing $D(R_i)$.

The student's modeling operation is matched stage by stage or instant by instant against the executions of $D(R_i)$ permitted in view of the student's operation up to the last instant and agreement is inferred if and only if there is at least one exact match.

In aggregate, these comments indicate the calibre of the conversational object language, L . It is, as previously stated, a language with a domain of interpretation (R and the modeling facility) and a pragmatic content (A and B have roles, such as teacher and student). It is a command and question language, not just an assertoric language; it has predicate variables their values being assigned by L users (in addition to fixed predicates), and it has temporal reference. The basic L statements are metaphors standing for analogies (rather than propositions standing for simple relations) and, operationally, agreement replaces veridical truth (though, as later, the notion of truth can be retrieved by a rather devious path).

Finally, L differs from the run of formal languages, in being interpreted or given a semantics with respect to a conversational domain which is so constructed that any relation R_i , in R can be retrieved, without loss of specificity from other relations R_j , R_l , in R using only operations that are themselves represented in R ; the representation $D(R)$ containing a grammar-like or permission-giving substructure ($D^1(R)$) that says how any R may be known or constructed in this manner. As a result, any topic relation learned legally is also reproducible and L reproducibility replaces the more familiar idea of L true or derivable.

⁵Similar comments apply to distinct fields predicated by the same individual; for example, the visual and the tactile fields of perception, in which the same figure may be given a visible and a tactile representation.

Individuals

Thus far we have assumed that a pair of individuals, A and B , could be distinguished apart without reference to the conversation in which they are engaged. Though plausible, this supposition cannot be upheld in practice. True, there are many ways of demarcating entities loosely called “individuals” (for example, by head counting) but when the entities are isolated they are not usually found to have the unitary or integral qualities required of A and B .

An external observer, looking on at an L conversation, can resort to many kinds of individuation. Two extreme methods are as follows.

To demarcate a processor, independently of the procedures it is executing. This is Mechanical Individuation or M Individuation since it resembles the isolation of a specific general purpose computing machine with a fixed spatio temporal location.

To demarcate a coherent cognitive organization or stable class of procedures, independently of the processors in which the procedures are executed. Such entities are called Psychological Individuals or P Individuals.

The most convenient definition proceeds thus:

Any strict conversation on domain R over occasion $0, 1, \dots, n, n + 1, \dots, N$ is a P Individual in its own right; moreover, it can be factored into a pair of entities A and B of which at least one (possibly both) are also P Individuals. To be noncommittal on this point, A and B are called participants. One of them (by convention) A is certainly a P Individual in the context of B, R . The other may be.

Due to the form of this definition, the P Individual has a certain primary. Its integrity as a P Individual is due to the fact that the procedures which make it up are self-reproducible in the conversational domain R . But they cannot in fact, be reproduced unless they are executed in an M Individual which is an L processor. Hence M Individuation is needed in order to talk about or set up a strict conversation, as well as P Individuation. It happens that P Individuals do not correspond, one to one, with distinct M Individuals unless special precautions are taken and the conversational milieu is specially designed (as it is in CASTE or as it may be in nonmechanical conversations like interviews and tutorials).

The phrase “self-reproducible” is used in the sense of the theory of self-reproducing automata. The reader may wish at this time to turn to Appendix A. It is crucial that a self-reproducing system can be recognized independently of the processor(s) in which it is being executed. In the case of strict conversations we use an equivalence, proposed by Loefgren, between explanations, modeling operations and reproductive operations, to recognize agreement over the explanation (modeling or reproduction) of a relation and which if conjoined with evidence for why or how the agreement was reached, signifies an understanding. The recognition of an understanding marks the end of an occasion during which the strict conversation was a self-reproducing system.

Regarding the processors that execute the procedures in a strict conversation, there are obviously constraints such as “any competent processor must be an L processor” as well as more specific limitations upon “compilers” for L procedures and the like. For brevity a P Individual’s processor must be compatible with the P -Individual. But if there is one

compatible processor then there may be indefinitely many compatible processors (as well as indefinitely many noncompatible processors).

A *P*-Individual is processor-independent insofar as it is immaterial which compatible processor executes the constituent procedures; it is immaterial whether the *P* Individual is executed in one processor or several; or whether several *P* Individuals are executed in the same processor. In fact a strict correspondence or even a strong correlation between *P* Individuals and their processors is seldom manifest and, as a rule the *P* Individual is distributed under execution. Further a *P* Individual (unless qualified in some way) has no necessary spatio temporal location. The *P* Individual is a collection of statements in the conversational object language *L* and shares the quality "lack of a spatio temporal locus" with any other class of *L* statements (of course an *L* statement may refer to time and place; further an utterance of the statement, one particular execution of a Proc, occurs at a specific time and place). The *P* Individual is properly specified with respect to a language (*L*) and a conversational domain (*R*), in which (whatever else it may be) the *P* Individual is self-reproducing. Interesting *P* Individuals do a great deal more than reproduce themselves; for example, they evolve in a conversational domain as a result of mutual understanding and this "evolution" is literally "learning."

Some familiar examples of *P* Individuals in other fields are as follows: A role in society (realized by any of a class of human beings); a character in a play (realized by any competent actor on a stage); a species (realized by any member organism).

Many sorts of *M* Individual can be distinguished by the methods of classical physics and behaviorism. Though a human brain is an obvious candidate for *M* Individuation, the fabric of an *M* Individual is not necessarily biological. "Mechanical Individual" carries no commitment on this issue (the brain for example, counts as a biological mechanism) and it is worth commenting that there is nothing in the specification of a *P* Individual to indicate that it must be executed by a brain.

Just as a *P* Individual is specified independently of the processors that execute it, so an *M* Individual is specified independently of the procedures it may execute (except for limitations which render it incompatible with certain procedures; in this respect it stands in the same relation to a procedure as a general purpose computer augmented by permanently attached compilers executive routines, etc. to its program).

Some uniformity is added by noting that an *M*-Individual is unitary, integral, and recognizable for precisely the same reason as a *P* Individual; namely, it is a self-reproducing system. Only, in this case, the replication processes are biological or contingent upon physical laws (determining the stability of materials) rather than symbol manipulating processes. Since they are independently identified, *M* Individuals and *P* Individuals belong to distinct ontological classes in Harre's sense, at any rate.⁶

⁶ Readers with an exclusive taste for monism may be disquieted by the suggestion of two (and if two, why not more) ontological categories. To reassure them, it is stated, once and for all, that the theory is formulated in respect of an external observer. Hence, the theory is in no essential way effected if the two entities "*M*-Individuals" and "*P*-Individuals" are replaced by the two observation processes, "*M*-Individuation" and "*P*-Individuation." For each occurrence of "*M*-Individual" it is legitimate to substitute "the observer's *M*-Individuation of *X*" and for each occurrence of "*P*-Individual" to substitute "the observer's *P*-Individuation of *X*" where *X* is the conversation under scrutiny. If the paper were written in that way it would, however, be a great deal lengthier. Hence, there is no necessary philosophical commitment to one position or the other. For my own part, I feel that a systematic dualism or pluralism is compatible

M Individuals differ according to the fabric of which they are made, their capacity for data storage and the arrangements for determining sequences of operations and overall control. All of these could be significant, but for the present purpose, the last type of constraint is of paramount importance, i.e. the arrangements for controlling the execution of processes. (See Appendix C.) We shall treat this constraint in terms of processor clocking and assign one processor clock to determine an execution sequence for each locus of control. These clocks (osillators driving shift registers if the reader prefers it) act as independent units, i.e. each locus of control represents an asynchronous automaton. But, depending entirely upon the procedure undergoing execution, the otherwise asynchronous loci of control may become synchronized by virtue of interactions prescribed by the procedures; not by the *M* Individual.

So, for example, a one clocked *M* Individual is a serial processor. A many clocked *M* Individual may accept several nonconflicting procedures and execute them simultaneously but without interaction; that is, "in parallel." On the other hand it may accept a procedure parts of which can be executed simultaneously such that the partial results of one computation constitute initial conditions for another; that is, it may compute "concurrently." The interactions that synchronize otherwise independent automaton (the loci of control) constitute information transfer; and only such synchronizing interactions convey information.⁷

The clocking of an *M* Individual characterizes the number of control loci it can accommodate and consequently the types of procedure it can execute (in particular, as mentioned in the modeling discussion, if a relation is taken as a verb or process then this number specifies its irreducible adicity). More generally any procedure whatever is placed in register with an *M* Individual (and thus executed) by establishing correspondence between the order of operations in the procedure and the beats of the processor clock(s). In particular these comments apply to classes of procedures such as *P* Individuals.

It will be convenient to distinguish the following categories of *M* Individual.

(a) *L* Processors that are compatible with and hence able to execute some specified *P* Individual in a given domain *R*.

(b) *L* Processors able to execute some *P* Individual but not necessarily compatible with a specific *P* Individual.

(c) Processors that are not *L* Processors; which may execute procedures but may not execute *P* Individuals.

(d) Storage media structured by "hardware" constraints (in the sense of data organization, word length, etc.).

Since an observable *P* Individual is a conversation involving participants *A* and *B* and since *A* and *B* are distinct apart from the information transfer due to the conversation it follows that *A* and *B* must have at least two loci of control so that any observable *P*

with scientific method in the broad sense, and am inclined to the view that it may lead to a neater picture of reality as well as a more condensed method of exposition.

⁷The usage in appropriate and in accord with Holt or Wattanabe. It does not, of course, correspond to "information transfer" in the technical sense of communication theory (Shannon, Gabor) or multivariate information analysis (Ashby, Garner, Hake) where (appropriately in a different context) the term refers to selection that reduces an observer's uncertainty or increases the specificity of his estimates.

Individual can only inhabit a many clocked processor. Scrutiny of Appendix A shows that a self-reproducing system is otherwise Turing representable and thus open to serial execution. It follows that any L processor (category (a) or (b)) is many clocked that a two-clocked processor is a candidate for inclusion in category (b) but that compatibility (and thus inclusion in category (a)) depends (amongst other things) upon there being sufficient processor clocks to accommodate the control loci required by a P Individual's procedures when executed in a specified conversational domain.

Conversely, a one-clocked processor cannot execute an observable P Individual. Although the restriction could have been imposed in other ways, we have specified a modeling facility as a processor that is not an L processor because it is rendered one clocked by the trick of stopping and zeroing the execution clock (τ) if the modeling clock (t) is started. This also (and equivalently) establishes the structure/function/distinction mooted earlier. Hence, as it stands, category (c) consists in one clock processors but (as later) this limitation is unnecessarily rigid.

Finally, we represent category (d) M Individuals (storage media) as processors that have as many asynchronous clocks as they have addressed positions; these clocks execute identity operations (one to each of the addresses) that rewrite the stored data. This image is devoid of any profound connotation; it is chiefly justified by convenience. By picturing storage in this way it is possible to regard the reading and writing operations that inscribe or delete data as synchronizing operations and/or to adjoin special clocks that allow us to instrument a recently proven equivalence between software (program) representations and hardware (storage organization) representations.

The categories (a), (b), (c), (d) of M Individuals feature in the theory as follows. L Processors (category (b)) are putative embodiments of P Individuals, conversations included. For a particular P Individual in a given domain an L processor may or may not be M Compatible (in Category (a)) and thus able to execute it.

Category (c) (processors that are not L Processors) serve as the paradigm modeling facility and have already been discussed; a facility limited to a model class M is simply a one-clocked processor inhabited by a procedure which provides the additional constraints required by M .

Category (d) processors typify the interface mechanism of CASTE. Recall that the interface contains a permanent inscription of the L descriptions $D(R)$ of the conversational domain. During any occasion, the participants A, B have order independent access to this data and, depending upon their activities, the modes that stand as place holders for topic relations R_i are marked by signal lamps indicating the values of predicates (aim, goal, understood and so on) that describe the current state of a node. The physical inscription of $D(R)$ together with the node marker values, is called the entailment structure; the physical inscription of $D^0(R)$ together with the modeling facility (in this case STATLAB) is called the task structure; between them, the entailment structure and the task structure are the interface which is embodied in an M Individual of Category (d). By electing to represent storage in a peculiar (and at first sight outlandish) way it is possible to speak, meaningfully, of synchronizing the interface with respect to two processors one of which is the student's brain and the other of which is the serial processor for executing the B heuristic. Consequently some of the students' procedures

(those of a P Individual A) may be executed in the interface and some of the B procedures may also be executed in the interface; to this extent a conversation itself a P Individual, is externalized. Stated otherwise, all information transfer due to the concurrent execution of A and B is mediated through the interface.

We now specify further the metalanguage (L^*) which is used for talking about conversations. The language is in the form of pictographs which describe permissible L transactions. We shall call these figures ikons. Though they resemble system flow charts certain very definite consequences follow from drawing them and they have the formal status of equations. They are made up of a very limited number of elements:

-  comparator, description symbol
-  task structure and entailment structure $D^0(R) + D^1(R)$
- or | (lines) are operators
-  direction signals
-  procedures Proc i or nameable collection of procedures
-  parametric operation carried out
-  provocative linkages, command and questions
-  causal linkages

A and B are prohibited from commanding or questioning themselves directly; if A “changes his attention,” for example, he must do so through transactions that lead B to issue the appropriate command. Because of this, reflexive commands of the kind Comm_{AA} or Comm_{BB} interpreted as A telling himself to do something and B telling himself to do something are not legal. A single subscript is sufficient and is used to designate the addressee. Thus Comm_A is a command addressed to A and Comm_B to B ; Comm_A is issued by B and Comm_B by A . Precisely the same commands apply to questions. More generally, by dint of these constraints all modeling/explanation cycles in a strict conversation pass through a vertical cleft which distinguishes A and B .

(c) This vertical cleft is denoted by “” (figure 9).

(d) The rule is that all connections penetrating “” alone are of provocative form as in figure 10.

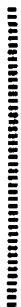


FIGURE 9. Vertical cleft.

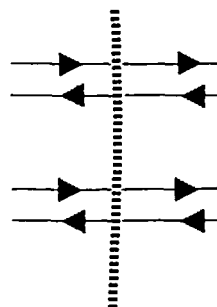


FIGURE 10. Provocative form of interaction.

(e) Equisignificantly, any transaction that crosses the vertical cleft is personally pronominalized as a command or a question (to you, or A or B) and any response or reply is tagged by its source (his or B 's or A 's).

A causal transaction is deductive and refers to it. In other words, the responsible participant (A , for example) entertains an hypothesis about how to model a relation R_i which he can instantiate and verify or deny, within the bounds of a modeling facility in a universe of his own choice, by forming a model $M_A i$. Two principal meanings can be ascribed to " A has an hypothesis"; namely, " A has a $\text{Proc}_A k, i$ yielding $M_A k, i$ and $\text{Proc}_A l, i$ believed to yield $M_A l, i = M_A k, i$ (the hypothesis that $\text{Proc}_A l, i$ is a concept for R_i is denied if $M_A l, i = M_A k, i$ is false)" or " A is given the name of R_i ; he produces (on command, Comm_A^1) the model $M_A i$, and this model either does or does not match a standard model M_i^* , inscribed in $D(R)$."

A provocative transaction is an explanation by A to B ; or alternatively a model submitted by A for B 's scrutiny (or vice versa). Since A 's universe of modeling may be distinct from B 's it follows that the confirmation/denial obtained in the causal case according to whether or not $M_A i = M_i^*$, is replaced by (at most) a criteria of A and B agreement; $M_A i \Leftrightarrow M_B i$ (for $M_A i$ submitted by A and $M_B i$ the model, perhaps M_i^* submitted by B). In general $M_A i \Leftrightarrow M_B i$ does not imply that A 's extension of R_i is the same as B 's extension of R_i .

Tests for the linguistic condition understanding are carried out by examining cycles of explanation severed by the vertical cleft symbol " $\equiv$ ".

A horizontal cleft symbol $\equiv$ indicates the distinction of domains.

A concept $\text{Proc } i$ is a procedure operating upon a domain of relations R_i ; a memory is a procedure (perhaps the same procedure) operating upon a domain of concepts. The concept and the memory are tagged according to their domain and thus kept distinct.

The distinction is due entirely to the external observer. There is no reason to suppose that cognitive systems really are hierarchical; it is simply convenient to describe them as though they are.

The distinction between causal and provocative linkage is given a special notation shown in figures 11 and 12. Any distinct entity (for example, one of the participants or

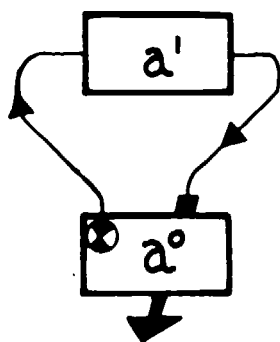


FIGURE 11. Causal coupling (a' has an hypothesis, models it in a° , checked description).

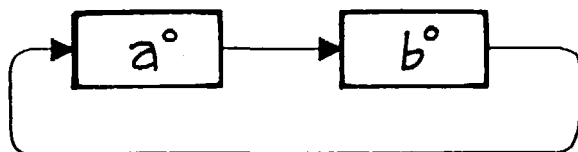


FIGURE 12. Provocative coupling (a° provokes or induces an action in b° and receives similar switching input from b°).

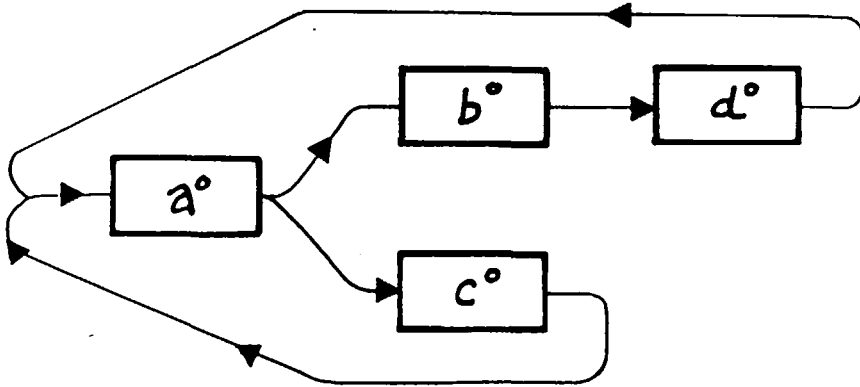


FIGURE 13. Sample network.

a particular concept Proc *i*) is enclosed in a box. In figure 11 a^1 acts causally upon a^0 through a cycle involving an operation (parametric arrow) and a description (the shaded comparator symbol). The meaning of figure 11 is that a^1 regards a^0 as an *it* about which a^1 has a deductive or inductive hypothesis and the expectation that this hypothesis will be confirmed after the operations are carried out (i.e. the description received will tally with the hypothesis). In the deductive case, only denial will inform a^1 since confirmation is anticipated). Any other cyclic linkage is provocative and is a search expected to furnish or to generate information. The simplest provocative linkage is shown in figure 12 but such cycles may bifurcate and rejoin, as in figure 13 and may also be combined (over the same set of boxes) with causal linkages (figure 14). The meaning of an entire (possibly complex) cycle is the demand for and the production of an explanation (or a class of them). Breaking the cycle at any one point (for example the output of a^1) the transaction may be interpreted, on the one hand, as "issuing a command" or "posing a

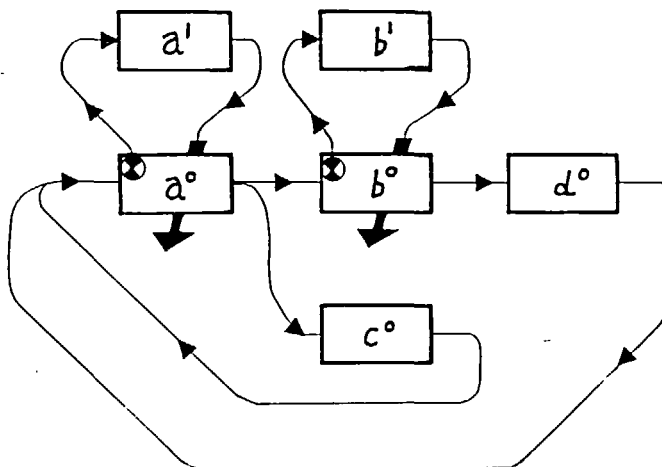


FIGURE 14. Network involving causal coupling and provocative linkages.

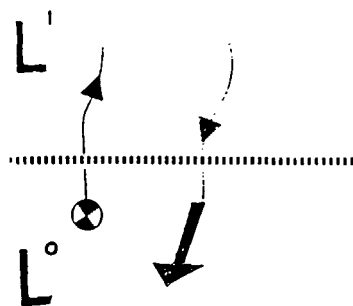


FIGURE 15. "Horizontal cleft" convention.

question (requiring an explanatory answer" or "presenting a choice amongst alternatives" (a question requiring a selection as an answer); on the other hand as "accepting a command in order to obey it" or "essaying an explanation" or "deciding." Notice however, that acts like "executing a command," "building a model," "giving an explanation" or "selecting a reply" involve causal transactions i.e., the doing agent has a hypothesis that whatever is done will, or is likely to achieve the desired result.

Causal cycles only can penetrate the horizontal cleft, figure 15. Equisignificantly, these are transactions exclusively concerned with "it." Quite possibly, A may regard a procedure in his own repertoire as "it" (and operate upon it). Or A may regard an environment like the modeling facility as it or he may regard B as an environment in which case the outgoing connections penetrate "...." first and later "..." also.

(d) The ruling is that for each participant A , and B there is at least one distinction of the type "...."; that for this pair of levels L' , L'^{+1} (by convention written L^0 , L^1

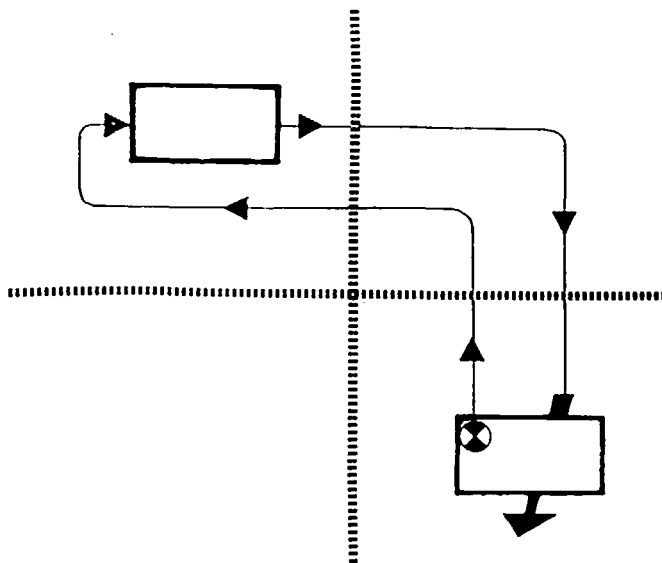


FIGURE 16. A disallowed causal primitive linkage.

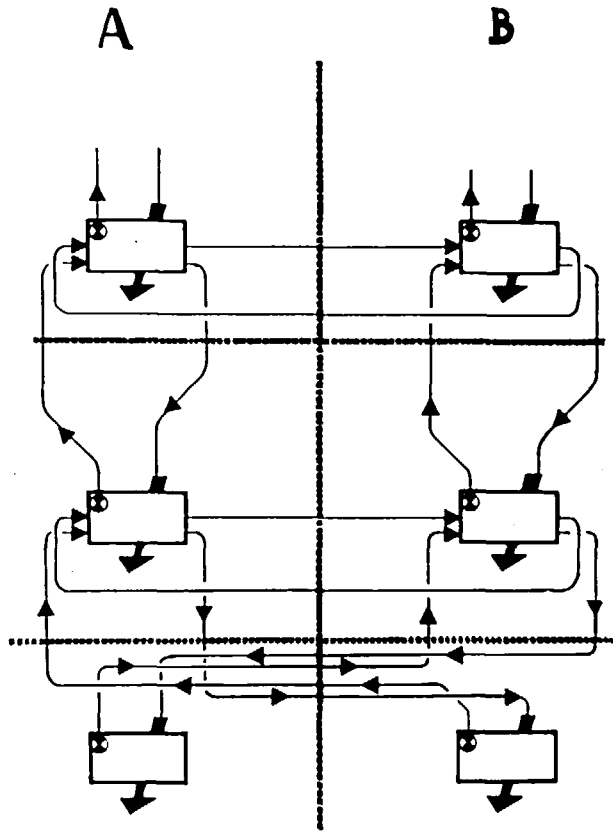


FIGURE 17. Permitted form of Fig. 16 construction.

though "0" and "1" have no more significance than " r " and " $r + 1$ ") there is at least one causal connection and that any causal connection at this level crosses no other boundary. For example, figure 16 is a forbidden construction. Figure 17 is not forbidden (and is interpreted as a causal linkage to an "it" other than "me") since elsewhere in the same construction there is (at least) one level at which the only caveat of (d) is satisfied.⁸ For example, on each side of " $\equiv$ " (in A or in B) a memory is a Proc acting causally on a domain of concepts (alias Procs) to rewrite or construct them.

(e) In any L conversation A and B are brought into proximity at the " $\equiv$ " cleft with at least one " $\equiv$ " cleft in register. Hence, the provocative L transactions penetrating " $\equiv$ " are stratified and L is said to have levels (of control or organization) designated $L = L^1$, L^0 . For example, A 's L^0 description of an environment, in terms of predicates of A 's selection, is at a "higher level" than the environment to which the description refers; A 's

⁸Several alternative connections yield the same essential constraint. For example, it is possible to introduce a minimal stratification L^2 , L^1 , L^0 . Similarly it is possible to distinguish the environment external to participants (that is, the environment in which this participant's procedures are not executed), as distinct from the environment constituting the participant's own processor. These alternatives are just as acceptable, but in practice, the convention selected is the most transparent.

description of A 's concepts (any one of which contains one of A 's L^0 descriptions) is at a "higher level" than the concept itself.

Equisignificantly, " Ξ " represents a boundary between two initially asynchronous systems A , B , that become synchronized because information is transferred by the A , B , dialogue (vice versa, the dialogue is due to synchronization).

(f) Because of that, it is meaningful and necessary to index transactions and procedures by a level-denoting superscript and by a participant-denoting subscript. So, for example, there are Proc_A^1 , Proc_A^0 , Proc_B^1 , Proc_B^0 , the same notation being applied to questions, commands, and descriptions.

(g) Combining the distinctions " $\dots$ " and " Ξ " together with the connection forms they permit, yields the ikon of figure 18. It is called the conversational tableau and is the least specific framework in which a strict conversation can be observed over just one occasion.

(h) The conversational domain R (that is, the topic of conversation), is accessible as an "it" to either participant. Since L is stratified ($L = L^1, L^0$) there are two descriptions of R (namely $D^1(R)$, $D^0(R)$ indicating what may be known and what may be done, respectively).

(i) Hence $D(R) = \langle D^1(R), D^0(R) \rangle$ is accessible at the proper level to A or to B . The cleavage planes needed to provide place holders for $D(R)$ (corresponding to the place holders for the Proc s of A and of B) are added to the tableau (figure 18) to yield a

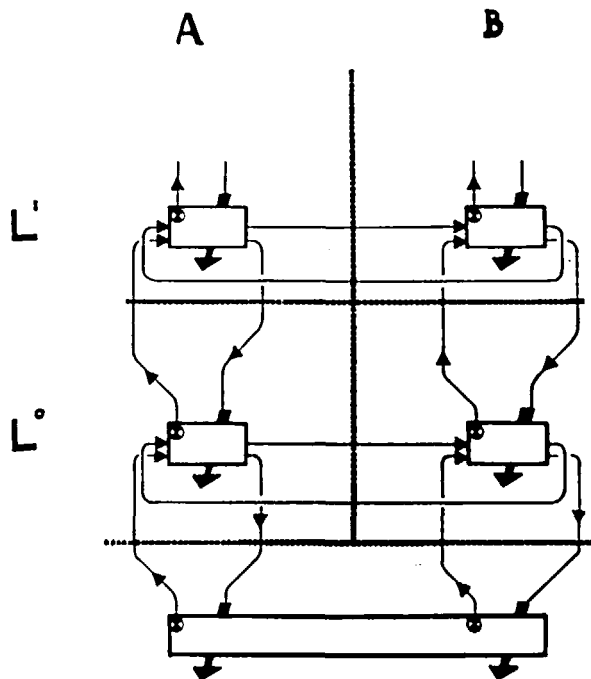


FIGURE 18. A tableau.

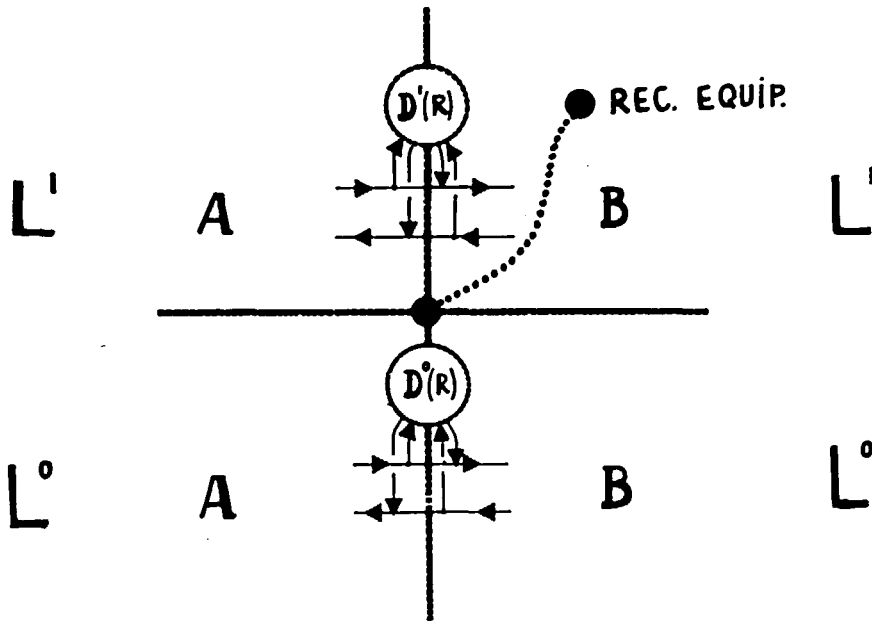


FIGURE 19. Conversational skeleton.

conversational skeleton (figure 19). This is the least specific structure in which it is possible to observe a conversation (anchored on domain R) over a series of occasions. To emphasize this point, the external observer's recording equipment is shown attached to the central cleft of the conversational skeleton.

The central cleft is a locus of understanding (involving at least two, usually more than two, loci of control). Hence, this is a locus (in the sense of a confluence of organizations) where the external observer can test for the condition of understanding and determine, thereby, the end of an occasion. Without further qualification the locus does not have a well defined spatio temporal position.

From clause (e) the locus of understanding is also a barrier marked out by the vertical cleft “ $\equiv$ ” across which information is transferred between two otherwise asynchronous systems A , B (each structured by the “ $\equiv$ ” cleft), so that A , B , become synchronized in conversation. Using “information transfer” in this sense, it can also be argued that the conversation is due to a complete or partial synchronization of A and B .

(j) Since the distinction of participants and the distinction of levels are introduced at the discretion of an external observer, there is no objection to adding more distinctions like them. The number is a matter of convenience. Parsimony alone dictates as few inscriptions as possible.

Properties of the Conversational Skeleton

Each place holder in the conversational skeleton (each quadrant in figure 19) can be filled either by a procedure, *Proc i* or a nameable collection of procedures and the special

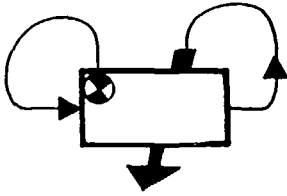


FIGURE 20. Trivial and disallowed construction.

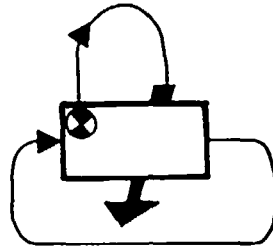


FIGURE 21. Another trivial and disallowed construction.

circular loci reserved for descriptions, can be filled by task structures ($D^0(R)$) and entailment structures ($D^1(R)$) for any conversational domain that is the context in which these procedures may be executed.

Some forbidden or trivial constructions are shown in figures 20, 21, 22. For example, figures 20 and 21 are unobservable because the asserted distinction between causal and provocative transactions is obliterated. In figure 22 it is impossible to impose an unambiguous cleft. Since causal and provocative transactions are confused the one cleft possible might either be “ $\equiv$ ” or “ $\equiv$.”

The construction in figure 23 is possible but formally disallowed as it eliminates the distinction between L^1 and L^0 where “1” and “0” are merely different indices, r and s , regardless of any ordering such as “ $r > s$.” In contrast, figure 24 is permissible on these grounds (for $L^r, L^s = 0, 1 \dots$ it is true that $L^r \neq L^s$), and the intransitivity is generally acceptable, i.e. ($L^2 > L^1$ and $L^1 > L^0$ do not necessarily imply $L^2 > L^0$). But a system of this kind, though it is held to exist, cannot be completely observed since it is impossible to sever all of the transactions between the subsystem (representing participant A) at an interface with participant B , involving just one distinction of level. An instance of the difficulty is shown in figure 25 where the uppermost procedure box is devoid of a specific domain if approximated to an interface of the type specified in the conversational skeleton. However, such systems are observable at more complex types of interface and, unlike the other “defective” constructions, do enter into the scheme of ikons.

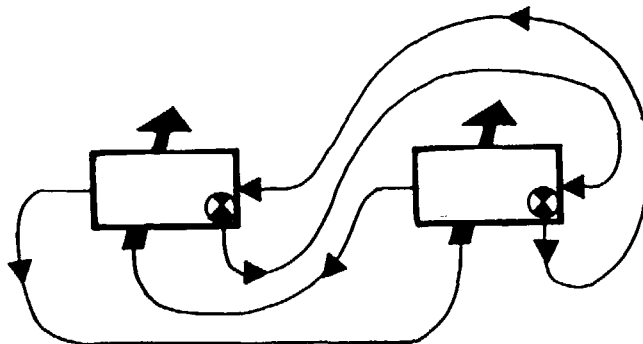


FIGURE 22. A new trivial but all the same defective scheme.

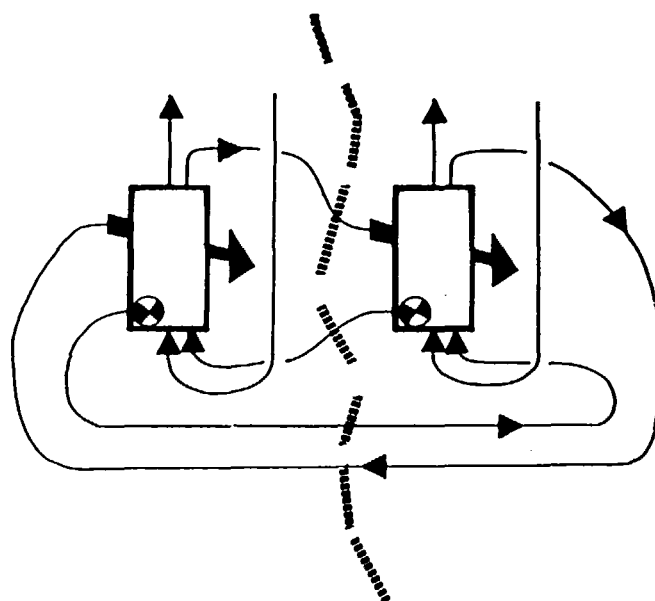


FIGURE 23. Possible but formally prohibited construction.

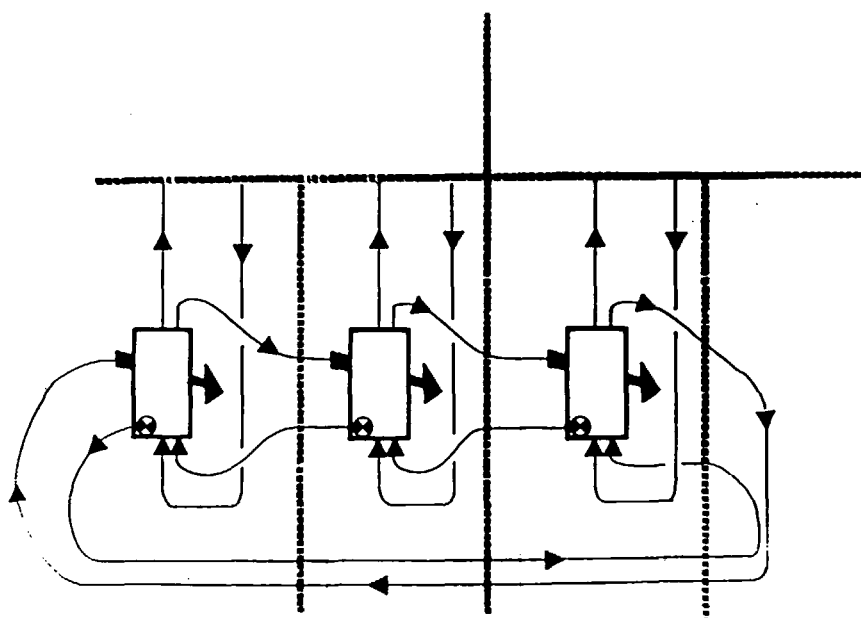


FIGURE 24. One existing but all the same unobservable scheme.

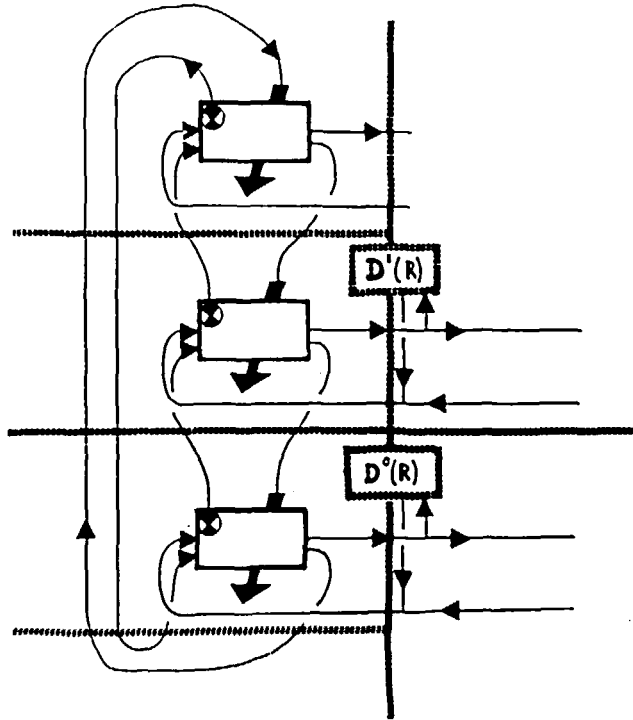


FIGURE 25. One case of the difficulty evident in Fig. 24.

Minimal Form of Conversational Skeleton

The conversational skeleton is the most parsimonious possible. It contains the structure needed if an external observer is to describe and discuss understanding and the general conduct of a conversation given that the participants are stable cognitive organizations and under the assumption that they furnish explanations in $L = L^1$, L^0 (as linguistic expressions) rather than building models and describing them. So, for example, the skeleton does not represent the use of STATLAB in CASTE, or any other modeling facility. Nor does it represent the reproductive processes that render A and B stable and (at least one) a P Individual.

In order to represent a modeling facility the external observer must introduce a further level of stratification, penetrated by causal connections that traverse Proc_A^0 or Proc_B^0 in the lowermost quadrants of the conversational skeleton. The L^0 dialogue coupling A and B thus includes L^0 descriptions of models themselves and of operations that act upon them. No further axioms are required since, according to the existing rules, any causal input must emanate from a description symbol “ $\otimes$ ” and the corresponding causal output must terminate on a parametric arrow symbol “ $\blacktriangleright$.” Since the descriptions refer to a facility constrained to accommodate only a certain class M , of models, A ’s description is X_A and B ’s description is X_B . Further A ’s parametric output is a selection from Y_A and B ’s a selection from Y_B . Figure 26 which shows the construction at issue is a

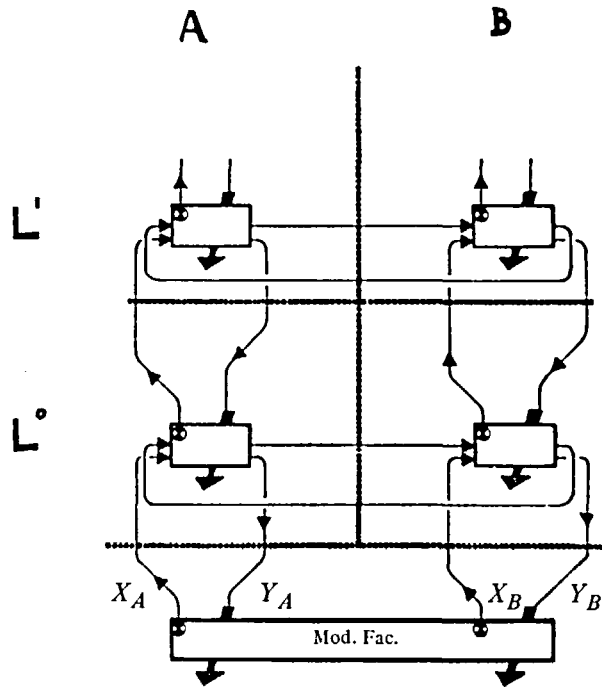


FIGURE 26. The canonical abstract construction and apart from labeling identical with Fig. 18.

labeling of figure 18. It is tempting to assign a level " L^{-1} " to the modeling facility. But this much is deliberately not given by the rules. These simply state that any model built in the facility has a level distinct from L^0 . It could, for example, be at level L^1 , just as well as " L^{-1} ."

The construction which makes an L^0 causal input a description of something at level L^1 and makes an L^0 causal output operate upon something at level L^1 is quite legitimate and in case the "somethings" described and operated upon are procedures Proc_A^1 located in the upper left quadrant of the conversational skeleton it means that some procedure Proc_A^0 is able to reproduce some procedure Proc_A^1 ; that is, to model it. If such a process took place for all procedures designated Proc_A^1 and if (referring to their input and output) they reproduced all procedures designated Proc_A^0 then the resulting cognitive organization would be stable. Though legitimate, we exclude this particular construction as unobservable in a strict conversation (it demolishes the hierarchy $L^1, L^0; 1 > 0$ or $L^r, L^s; r > s$ which is needed within the conversational skeleton in order to recognize understanding). However, very similar constructions are employed in the theory to obtain the same result in an observable fashion.

Two constructions of this type are particularly important.

(a) Since the hierarchy within the conversational skeleton is peculiar to a participant A or B it is not destroyed by the coupling which makes Proc_A^0 model Proc_B^1 and vice versa Proc_B^0 model Proc_A^1 . The specificity of level lost in the coupling can be retrieved by recourse to the distinction between the participants, provided that A and B are stable

cognitive organizations that remain recognizable over time. This will be true if the coupling has the effect mooted in the last paragraph (namely that all procedures in all of the boxes reproduce all of their subordinates). Notably the satisfaction of these conditions does not preclude the possibility that the Proc *s* in question may do other things as well as reproducing existing Proc *s*.

(b) In construction (a) there is no objection to assigning the coupling whereby Proc_A⁰ models Proc_B¹ and Proc_B⁰ models Proc_A¹ to a higher level, L^2 (a level "above" Proc_A¹ and Proc_B¹). The aptness or otherwise of this usage depends entirely upon the embodiment of the procedures under discussion; their execution in a processor. For example, it seems natural to regard a modeling facility as belonging to a level "lower" than L^0 because STATLAB and its kin are tangible devices. By the same token it seems natural that the same interactions belong to a "higher" level if they constitute *A*'s attempts to model *B*, or, vice versa, *B*'s attempts to model *A*. Here participant *A* operates upon whatever *L* processor executes participant *B* and conversely *B* operates upon whatever *L* processor executes *A* (remember that a modeling facility like STATLAB, regarded as an *M* Individual, is a processor but is not an *L* processor). From the present point of view, either "higher or lower" is equally valid. How natural the usage is depends upon the *M* Individuation of participants, of modeling facilities and the like, in particular kinds of fabric.

The notation used for ostension is shown in figure 27, a filling out of the conversational skeleton to produce an ikon for the condition that R_i is understood by the participants *A* and *B*. To image ostension R_i (rather than R) is entered in the compartments reserved for $D^1(R)$ and $D^0(R)$, an arc is drawn from $D^0(R)$ to the parametric or causal inputs of *A* and *B* (the boxes containing Proc_A⁰ and Proc_B⁰ *i*) to represent the "*A*! attend to R_i " or the "*B*! attend to R_i " part of a command or question. In contrast, the provocative coupling between *A* and *B* at level L^0 represents the problem solving activity engendered by the command or question in respect of R_i and this provocative transaction takes place in the context of data regarding how R_i may be brought about (shown as arcs at the appropriate level entering or leaving $D^0(R_i)$). The same convention is employed to depict ostension for transactions taking place at L^1 (here, the connections enter or leave the compartment filled by $D^0(R_i)$). If there is no subscript (so that $D(R) = \langle D^1(R), D^0(R) \rangle$) then the participants are at liberty to ostend any topic relation R_i in R_i .

The ikon in figure 27 also represents a modeling facility which may be connected to the participants in the manner of figure 26. However, the connections are shown as thin dotted lines as they are optional. The optionality is due to the fact that all of the conditions under discussion at level L^0 can be realized by question/explanation/transactions only. If a model is made, its construction is interpreted as a practical explanation; for generality, the argument is phrased as though a modeling operation is always required and if that is actually the case then the connections to the modeling facility must be established.

With this preamble, we shall fill the conversational skeleton in figure 27, figure 28, figure 29, figure 30, to depict agreement over and holding of a concept of R_i ; agreement and possession of a memory of R_i ; the condition of understanding; the same in respect of a conversational domain; the process of learning and a characterization of *P* Individuals.

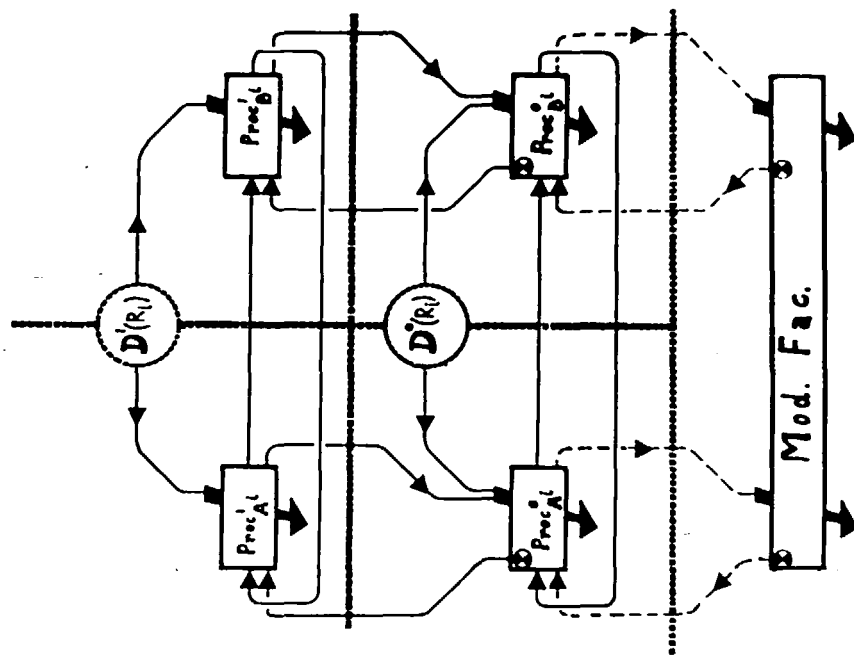


FIGURE 27. Notation employed for ostending or pointing at a topic.

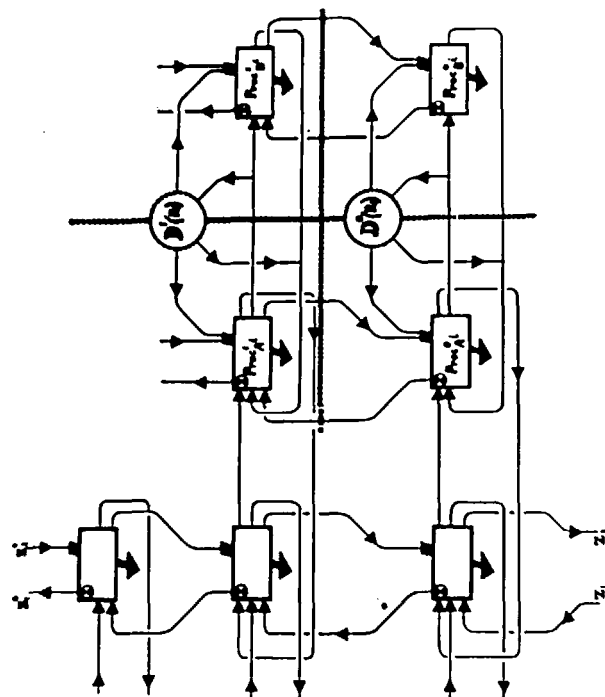


FIGURE 28. Integrated construction.

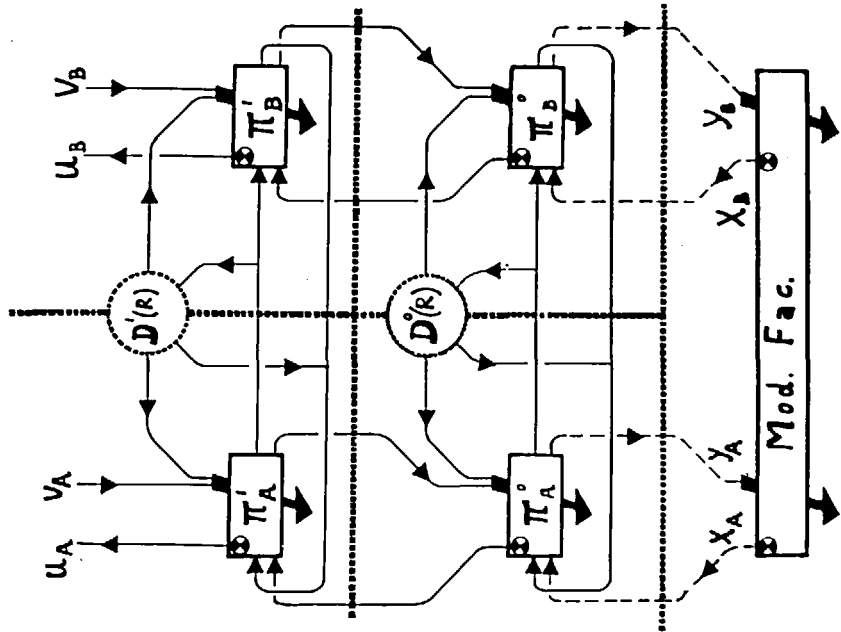


FIGURE 29. Notation for general reference to topics in contrast to ostending a particular topic (and thus for attention directing transactions).

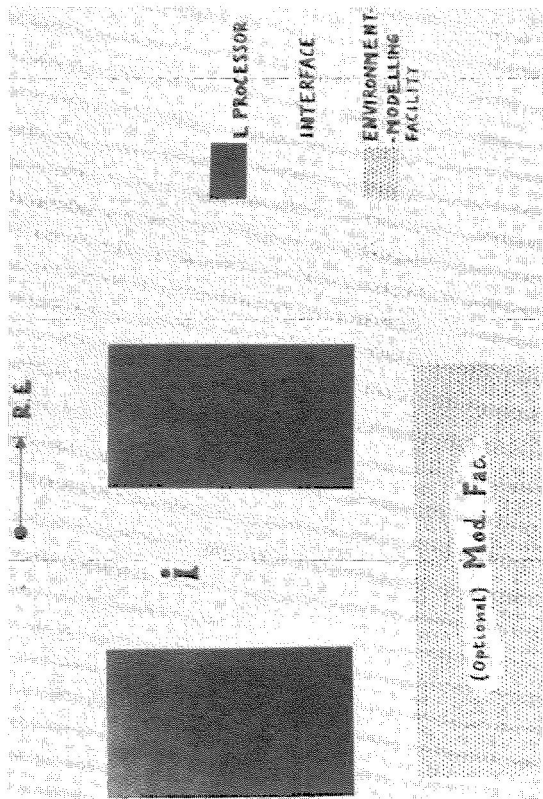


FIGURE 30. Notation for disconnecting types of M Individual.

A concept of topic relation R_i in a P Individual A is a class of procedures $\text{Proc}_A^0 i$ that reproduces R_i as it is known by A (namely as a particular description $D_A^0(R_i)$). Thus

$$\text{Proc}_A^0 i (R_i) \Rightarrow \langle \alpha^0, \beta^0, \dots R_i \rangle = D_A^0(R_i)$$

where α^0, β^0 , are L^0 expressions, the first being axioms in L^0 ; all of them in $D^0(R_i)$.

$$\text{Proc}_A^0 i (X) \Rightarrow \langle a^0, b^0, \dots M_A i \rangle = \text{Exec}_A^0 i$$

where a^0, b^0 , are partial models at instants t in modeling $X_t = \{x\}_t$.

Further, $\text{Proc}_A^0 i (M_A i) = M_A i$ and $D_A^0(R_i) \Rightarrow M_A i$ (if the modeling facility is restricted in the manner of figure 1).

A specific or qualified concept (A 's i th concept of R_i) is a serial procedure $\text{Proc}_A^0 l, i$ in $\text{Proc}_A^0 i$.

The execution of $\text{Proc}_A^0 i$ is interpreted as the execution of any or all of the $\text{Proc}_A^0 i$; if several elementary procedures are executed then the execution is concurrent; the unqualified execution of $\text{Proc}_A^0 i$ is interpreted straightforwardly as the concurrent execution of all procedures or of the concept as a whole. This yields $D_A^0(R_i)$. If the modeling facility were an L processor (it is not, by edict) then $D_A^0(R_i) \equiv \{M_A i\}$. Due to modeling facility restrictions only some (perhaps one) $M_A i$ is actually built.

Suppose that distinct participants A and B ostend R_i an occasion n and that they agree about their concept of R_i . That is $D_A^0(R_i) \Leftrightarrow D_B^0(R_i)$ and, if a model is actually built, $M_A i \Leftrightarrow M_B i$.

Agreement over a topic relation R_i is pictured in figure 27 by ostension and the provocative coupling that joins $\text{Proc}_A^0 i$ to $\text{Proc}_B^0 i$ at level L^0 . Model building is optional.

The coupling is secured by a vertical cleft. Consequently it can be represented by a command and question transaction between the participants. The symbol " $\rightarrow$ " denotes "provokes in the addressee" and the transactions will be expressed one sidedly in terms of commands/questions/addressed by B to A which elicit responses/replies/from A to B .

The Base Command and the unqualified question form is as follows.

$$\text{Comm}_A^0 i = \langle A! R_i / \text{Precon}^0 i \rangle \rightarrow \text{Proc}_A^0 i (X) = \text{Exec}_A^0 i$$

where $\text{Precon}^0 i$ is the modeling facility.

$$\text{EQuest}_A^0 i = \langle A? \text{Expl}^0 / R_i \rangle \rightarrow \text{Proc}_A^0 i (R_i) \Rightarrow D_A^0(R_i) = \text{Expl}_A^0 i$$

where the precondition is the statement of R_i in $D^0(R_i)$. If the modeling is unrestricted $\text{Expl}_A^0 i \equiv \text{Exec}_A^0 i$ (Appendix A); if not then the terminal model is $M_A i$.

The qualified command and question forms are

$$\text{Comm}_A^0 l, i = \langle A! R_i / \text{Precon}^0 i \wedge \text{Stat } l \rangle \rightarrow \text{Proc}_A^0 i (X) \Rightarrow \text{Exec}_A^0 l, i$$

in which the terminal model is $M_A l, i$.

$$\text{EQuest}_A^0 l, i = \langle A? \text{Expl}^0 l / R_i \rangle \rightarrow \text{Proc}_A^0 l, i (R_i) \Rightarrow \text{Expl}_A^0 l, i$$

Rewriting these expressions as commands or questions that are addressed by A to B and noting (initial supposition) that A and B agree (so that $D_A^0(R_i) \Leftrightarrow D_B^0(R_i)$) yields a symmetrical system, which is the L^0 coupling in figure 27.

In general, reproduction is designated Rep so that

$$\text{Concept}_A i \triangleq \text{Rep}_A(R_i)$$

or preserving level

$$\text{Concept}_A i \triangleq \text{Rep}_A^0(R_i)$$

A memory of topic relation R_i in a P Individual A is a class of procedures that reproduces some or all of A 's concept of R_i , namely $\text{Proc}_A^1 i$. Since the execution of $\text{Proc}_A^1 i$ takes place in an L processor that also embodies $\text{Proc}_A^0 i$ (not in a modeling facility) the stages in the concurrent execution series are not necessarily ordered. However, in conformity with the previously established notation they are represented as though they were (keeping the fiction in mind). The result of execution is a dynamic model in an L processor, thus a replica of $\text{Proc}_A^0 i$ executed by this L processor.

$$\text{Proc}_A^1 i (\text{Proc}_A^0 i) \Rightarrow \langle a^1, b^1, \dots, \text{Proc}_A^0 i \rangle = \text{Exec}_A^1 i \equiv D_A^1(R_i).$$

Where $D_A^1(R_i)$ is A 's explanation of how he constructed the procedures in $\text{Proc}_A^0 i$.

A general memory may be qualified (that is, A can usually reconstruct $\text{Proc}_A^0 i$ by several methods). The k th method is $\text{Proc}_A^1 k, i$ ($\text{Proc}_A^0 i$) $\Rightarrow \langle d^1, c^1, \dots, \text{Proc}_A^0 i \rangle = \text{Exec}_A^1 k, i$ such that $D_A^1(R_i) \Rightarrow \text{Exec}_A^1 k, i$.

A general memory can be applied to a qualified concept of R_i ; $\text{Proc}_A^1 i$ ($\text{Proc}_A^0 l, i$) $\Rightarrow \langle e^1, f^1, \dots, \text{Proc}_A^0 l, i \rangle = \text{Exec}_A^1 l, i$ such that $D_A^1(R_i) \Rightarrow \text{Exec}_A^1 l, i$.

A qualified memory (if it exists) can be applied to some (but not all) qualified concepts. If $\text{Proc}_A^1 k, i$, is distinct from $\text{Proc}_A^1 i$ then it can be applied to at least one concept: generally there are specific pairs k, l for which the following operation is executed.

$$\text{Proc}_A^1 k, i (\text{Proc}_A^0 l, i) \Rightarrow \langle h^1, g^1, \dots, \text{Proc}_A^0 r, i \rangle = \text{Exec}_A^1 l, r, i$$

The index, r , of the production, may or may not equal the qualifying index of the original concept. Empirically it is rare to find $\text{Proc}_A^0 l, i = \text{Proc}_A^0 r, i$. But both qualified concepts do belong to the concept of R_i ; that is $\text{Proc}_A^0 l, i \in \text{Proc}_A^0 i$ and $\text{Proc}_A^0 r, i \in \text{Proc}_A^0 i$.

The general form of a memory is

$$\text{Memory}_A i \triangleq \text{Rep}_A (\text{concept}_A i) \triangleq \text{Rep}_A^1 (\text{Rep}_A^0 (R_i))$$

which preserves the level superscript, of necessity.

This process is represented by the casual coupling in the left hand (or *A*) part of figure 27 (and $\text{Rep}_B^1 (\text{Rep}_B^0 (R_i))$ by the right hand (or *B*) part.

If distinct participants agree about their memories of R_i (thus, $D_A^1 (R_i) \Leftrightarrow D_B^1 (R_i)$), this fact is represented by the cyclic provocative coupling in the upper or L^1 part of figure 27.

Since the cycle is severed by the vertical cleft it can be represented by a command and question transaction whereby *A* tells *B* to reconstruct his concept of R_i or asks *B* how he does reconstruct his concept of R_i (by symmetry, also, whereby *B* tells or asks *A* to do the same thing). This command (to reconstruct, recall or reproduce) at least one concept of R_i has, as its precondition, the existence of *L* Processor models (i.e. concepts) for R_i ; that is of $\text{Proc}_A^0 i$ and $\text{Proc}_B^0 i$. Since $\text{Proc}_A^0 i$ and $\text{Proc}_B^0 i$ exist in an *L* processor not in a modeling facility, they are not directly observable at an interface between *A* and *B*. But the L^1 explanations that constitute replies to EQuest_A^1 , or EQuest_B^1 , (table 2) are observable as *L* transactions (saying how *A* or *B* reproduce their concepts of R_i). These transactions are detailed below in the context of a broader category; namely, the transactions characteristic of learning.

The L^1 analogue of the L^0 constructive operation $\text{Proc}_A^0 i (X)$ (in contrast to a reproductive operation, or memory) is an act of learning a concept. To exhibit the process in isolation it is necessary to consider a situation in which participant *A* does not have a concept $\text{Proc}_A^0 i$ for R_i but does have concepts for all R_{jk} that occupy nodes $j_i \dots j_{mk}$ in at least one of the *Im Ent Set* k_i of node *i* (occupied by R_i which is ostended on the occasion in question). In the CASTE facility this condition is obtained if node *i* is not marked as being understood but is ostended whereas all nodes in at least one *Im Ent Set* k_i are marked understood.

A legal learning operation (i.e. a learning operation that can be interpreted in that facility) is

$$\text{Proc}_A^1 i (\langle \text{Proc}_A^0 j_i \dots \text{Proc}_A^0 j_{mk} \rangle) \Rightarrow \langle r_1, s_1 \dots \text{Proc}_A^0 i \rangle$$

Where legal and thus interpretable concurrently occurs where $\text{Proc}_A^1 i$ operates upon a class of ordered preconditions corresponding to *Im Ent Set* $k_1 i$, *Im Ent Set* $k_2 i$, and so on.

This operation is also represented by the left hand causal connection of figure 27 or (in the case of *B*) by the right hand causal connections of figure 27.

The L^1 transactions of learning are obtained by severing the provocative coupling in the upper or L^1 part of figure 27 for the case in which the precondition is $\langle \text{Proc}_A^0 j_i \dots \text{Proc}_A^0 j_m \rangle$ (or its generalization to a class of preconditions) rather than $\text{Proc}_A^0 i$. The transactions are represented by the command and question forms of table 2 and are shown below in terms of commands or questions addressed by *B* to *A* which meet with responses or replies from *A* to *B*. The responses, being constructive operations in an *L* processor, are not directly observable at an interface but the explanations which

constitute replies to the questions are observable as part of the L^1 dialogue and are one segment of a learning strategy as in plates 6 and 7.

The transactions may either be unqualified or qualified.

The unqualified transaction forms are

$$\text{Comm}_A^1 i = \langle A!, R_i / \text{Precon}_i^1 \rangle \rightarrow \text{Proc}_A^1 i (\text{Precon}_i^1) \Rightarrow \langle r^1, s^1, \dots, \text{Proc}_A^0 i \rangle = \text{Exec}_A^1 i \equiv D_A^1(R_i)$$

where Precon_i^1 is either $\text{Proc}_A^0 i$ (in case $\text{Comm}_A^1 i$ is a command to reconstruct) or $\langle \text{Proc}_{Aji}^0 \dots \text{Proc}_{Ajmk}^0 \rangle$ (in case $\text{Comm}_A^1 i$ is a command to learn).

$$\text{EQuest}_A^1 i = \langle A? \text{Expl}_i^1 / R_i \rangle \rightarrow \text{Proc}_A^1 i (\langle \alpha^1, \beta^1, \dots, R_i \rangle) = \text{Expl}_A^1 i \equiv D_A^1(R_i)$$

where $\alpha^1, \beta^1, \dots$ and so on are L^1 expressions comprehended by $D^1(R_i)$ (just as α^0, β^0 , are comprehended by $D^0(R_i)$). They may either be parts of A 's explanation of how he constructed $\text{Proc}_A^0 i$ (in case $\text{EQuest}_A^1 i$ asks "how did you recollect R_i ?") or data obtained as information from $D^1(R_i)$ (in case $\text{EQuest}_A^1 i$ asks "how do you learn a concept for R_i ?").

The qualified transactions are as follows:

$$\text{Comm}_A^1 ki = \langle A!, R_i / \text{Precon}^1 ki \rangle \rightarrow \langle p^1, q^1, \dots, \text{Proc}_A^0 ki \rangle = \text{Exec}_A^1 k, i;$$

A command to build a concept of R_i from the specific precondition $\langle \text{Proc}_{Aji}^0 \dots \text{Proc}_{Ajkm}^0 \rangle$ or to reconstruct it by a specific method.

$$\text{EQuest}_A^1 k, i = \langle A? \text{Expl}^1 ki / R_i \rangle \rightarrow \text{Proc}_A^1 i (\langle \text{Proc}_A^0 ki \rangle) \Rightarrow \langle \alpha^1, \beta^1, \dots, \text{Proc}_A^0 ki \rangle = \text{Expl}_A^1 k, i;$$

Namely, "how (specifically) do you learn your concept of R_i on this occasion" or (depending upon the precondition) "how did you reconstruct your concept of R_i on this occasion?"; both are transactions at level L^1 of L .

The unqualified commands and questions may refer through a specific $\text{Proc}_A^1 l, i$ to the general concept of R_i (namely $\text{Proc}_A^0 i$) by directives of the form:

$$\text{Comm}_A^1 l, i = \langle A! R_i \wedge \text{Stat } l / \text{Precon}^1 i \rangle$$

or

$$\text{EQuest}_A^1 l, i = \langle A? \text{Expl}^1 i / R_i \wedge \text{Stat } l \rangle$$

to yield the construction or the explanation of how to construct $\text{Proc}_A^0 l, i$ so that $D_A^1(R_i) = \text{Expl}_A^1 l, i$ and describes $\text{Expl}_A^1 l, i$. Thus, psychologically, you may be asked to learn a particular concept in any way, or to explain how you learned it (failure to do so will evidence your inability to entertain the specific concept not your inability to learn it); similarly, if you have a specific concept you may be asked to reconstruct it in any

manner or to say how you did so (failure will evidence falsity of the supposition that you had the specific concept in the first place).

In contrast, the qualified L^1 commands and questions may only be asked of certain definite kinds of specialized concept: that is only certain pairs l, k , are compatible. Using the subscript indices, the result of demanding a reconstruction may be the same specialized concept ($\text{Proc}_A^0 l, i$) or a possibly different specialized concept ($\text{Proc}_A^0 r, i$; where r is not necessarily the same index as l) or no concept whatever.

If the result is $\text{Proc}_A^0 l, i$ then the pair of procedures $\text{Proc}_A^1 k, i$ and $\text{Proc}_A^0 l, i$ are called P/P compatible in the context of R_i ; if the result (for r other than l) is $\text{Proc}_A^0 r, i$ then $\text{Proc}_A^0 l, i$ is mutable under $\text{Proc}_A^0 k, i$ in the context of R_i ; and, if the result is no concept, then $\text{Proc}_A^1 l, k, i$ and $\text{Proc}_A^0 l, i$ are said to be P/P incompatible in the context of R_i .

The same comments and definitions apply to the explanations elicited by a qualified question, asked in respect of a specific concept and a given topic relation.

Psychologically, even if you do have a specific concept, by token of which you can reconstruct it somehow and give an account of your reconstruction, it does not follow that you can reconstruct it by a particular method even though you may also give an account of the method, and use it with respect to some other concept. If you can do so, then the concept and the method are P/P compatible; if not, they are P/P incompatible; if a fresh concept is produced as a result, then the original concept is mutated by the method.

If the participants A and B , ostending topic relation R_i on occasion n are in agreement about their concepts of R_i and if they are also in agreement about their memories of R_i , that is, about how to reproduce their mutually agreed (not identical) concepts then they agree about stable concepts of R_i and are said to (mutually) understand R_i . If so, in the CASTE facility, the node i of R_i is marked as understood on occasion n .

The condition of understanding R_i is represented by the complete ikon of figure 27 and is one way of filling the conversational skeleton.

This ikon says more than the equations taken in isolation. It says that the system of equations is stable under execution in an appropriate M Individual.

It represents the sprout or growing point of a strict conversation and is anchored on the topic relation (R_i) ostended on the n th occasion.

If (as is often prescribed in the sequel) the L^1 procedures of participant B are the execution of $D^1(R_i)$ and the L^0 procedures of the same participant are the execution of $D^0(R_i)$, then an understanding becomes, at the sprout, a correct understanding.

That is, an agreed L^0 explanation (or model) $D_A^0(R_i) \Leftrightarrow \text{Some } D^0(R_i)$ is a correct explanation (or model), in the sense that it satisfies what may be done; ($D^0(R)$).

Similarly, an agreed L^1 explanation $D_A^1(R_i) \Leftrightarrow \text{Some } D^1(R_i)$ is a correct explanation in the sense that it satisfies what may be known; ($D^1(R)$).

These comments apply, directly to A but are symmetrical. It is more usual to call the B component ($\text{Exec}_B^0 i$) a demonstration (by modeling) or a demonstrative explanation (taken as a stretch of L^0 dialogue) of R_i .

Similarly the B component ($\text{Exec}_B^1 i$) is called advice on how to learn if it is tendered permissively, or a teaching strategy, if it is a mandate.

Whenever it is prescribed that $\text{Proc}_B^1 \equiv D^1(R_i)$ and that $\text{Proc}_B^0 \equiv D^0(R_i)$, this formula applies to all R_i in R . Hence for any locus of the sprout of a conversation on "understanding" is a correct understanding.

It is legitimate to augment a construction like figure 27 by writing R in place of R_i (so that the sprout of the conversation may rest on any topic relation in R) and by introducing any legal proliferation of process boxes and connections that retains stable understanding of whatever R_i in R is ostended on a given occasion. For example, figure 28 is a legitimate ikon. Since the connections around the central cleft assert that the system maintains understanding at the central cleft, restrictions are thereby imposed upon the contents of the unlabeled process boxes in participant A . Such proliferated constructions are henceforward given the generic label Π if they are, or form part of, a (self-reproducible) P Individual.

One obvious way to obtain a Π that is a P Individual is to extend the general representation of concepts, memories, etc. by one further stage. That is, to move from

$$\text{Concept}_A i \triangleq \text{Rep}_A^0(R_i)$$

$$\text{Memory}_A i \triangleq \text{Rep}_A^1(\text{Rep}_A^0(R_i))$$

to

$$P \text{ Individual} \triangleq \text{Rep}(\text{Memories}(R_i))$$

The following constructions are legitimate for any P Individual A in domain R_i or R

$$\text{Rep}_A^0(\text{Rep}_A^1(\text{Rep}_A^0(R_i)))$$

For all R_i in R ;

$$\{\text{Rep}_A^0(\text{Rep}_A^1(\text{Rep}_A^0(R_i)))\}$$

in the sense that the general "Rep" can be substituted by Proc_A^0 and Proc_A^1 that execute it; (for example, reproductive tessellation automata, populations of reproductive automata). But the reproductive process (a kind of "self understanding" perhaps) cannot be observed at a central cleft; hence, it cannot be detected as an event called understanding at an interface; hence, there is no way to demarcate the occasions of a strict conversation. As an orienting comment it is possible to detect understanding if the specificity of level (that is lost by allowing Proc_A^0 realizing Rep_A^0 to act upon Proc_A^1 realizing Rep_A^1 as well as vice versa) is retrieved by a distinction of subscripts. Thus for all R_i in R ;

$$\{\text{Rep}_B^0(\text{Rep}_A^1(\text{Rep}_A^0(R))) \text{ and } \text{Rep}_A^0(\text{Rep}_B^1(\text{Rep}_B^0(R_i)))\}$$

is satisfactory in this respect, if the subsystems are interlocked for example by

$$\text{Rep}_A^0(\text{Rep}_B^1(\text{Rep}_B^0(\text{Rep}_A^1(\text{Rep}_A^0(R_i)))).$$

The canonical ikon is shown in figure 29. The entire conversation is a P Individual in the context of R . As it stands, any one of the participants also might be a P Individual (by back coupling the connections X_A, Y_A to U_A, V_A or X_B, Y_B to U_B, V_B ; perhaps

though a nexus of the kind in figure 28). By definition, these reproductive couplings may not be observed but an alternative (again possibly indirect) connection can be observed (namely; X_A, Y_A to U_B, V_B and X_B, Y_B to U_A, V_A) if a further level of discourse (generally though improperly called L^2) is added. Alternatively X_A, Y_A, X_B, Y_B may penetrate a modeling facility liberal enough to act as an L processor. But this possibility is prohibited deliberately. There is also a sense in which, if A and B are in continual agreement, if B is designed so that Proc_B^1 in Π_B^1 represents the execution of $D^1(R)$ and Proc_B^0 in Π_B^0 of $D^0(R)$ (so that any agreement is correct agreement) then B can act as a surrogate for the replicative processes in A and, further, ensure that both construction and reproduction are correct or legal.

Figure 30 is the ikon for representing M Individuals independently of the procedures they execute. Different M Individuals are depicted by distinct blocks; different kinds of M Individuals are indicated by shading; the interaction between M Individual is shown by juxtaposition.

For the present discussion (as before) it is only essential to distinguish the following kinds of M Individuals.

- (a) Compatible L processors that execute concurrent procedures; these are labeled α, β .
- (b) An interface, ι , also many clocked, that can act as a storage medium embodying the "hardware constraints" of computation theory and also as a limited L processor. $D(R) = \langle D^1(R), D^0(R) \rangle$ is embodied in the interface as the pair $\langle \text{Entailment Structures, Task Structure} \rangle$.
- (c) A modeling facility either one clocked (figure 7) or many clocked (figure 8) constrained to execute models of a class, M , that is prespecified.

To represent the execution of P Individuals at a definite place and time, the procedural ikons developed up to figure 30 can be imposed upon the M Individuals of figure 30. The combination is permissible if the M Individual on which a procedure is superimposed is able to execute this procedure (for example, if a P -Individual, A , is to be executed, locally, in one M -Individual, then A can only be executed in α or β ; as a convention A is assigned to α).

Figure 31 is the unique superimposition of figure 29 (stripped of modeling facility) upon figure 30

Figure 32 is the unique superimposition of figure 29 (augmented by its modeling facility) upon figure 30

By one means or another the entire conversation is a P Individual in the context of R . From appendix A, this P Individual may be factored into components A and B which are productive but not necessarily reproductive in B , such that one of them (A) is a self reproducing P Individual in the context of B, R and the other (B) may or may not be.⁹ Special interest is attached to the circumstance under which one factor (B) is not self-reproducible (hence not a P Individual) and the other (A) though self-reproducing in

⁹ The case in which neither factor is reproducible in R is excluded for psychological reasons only. A pair of factor subsystems S_A, S_B (rather than Π_A, Π_B) have inherent interest if their combination is a P individual in the domain R . (We shall use this construction, with certain added connections, and the evolutionary form of domain $R = R(n)$).

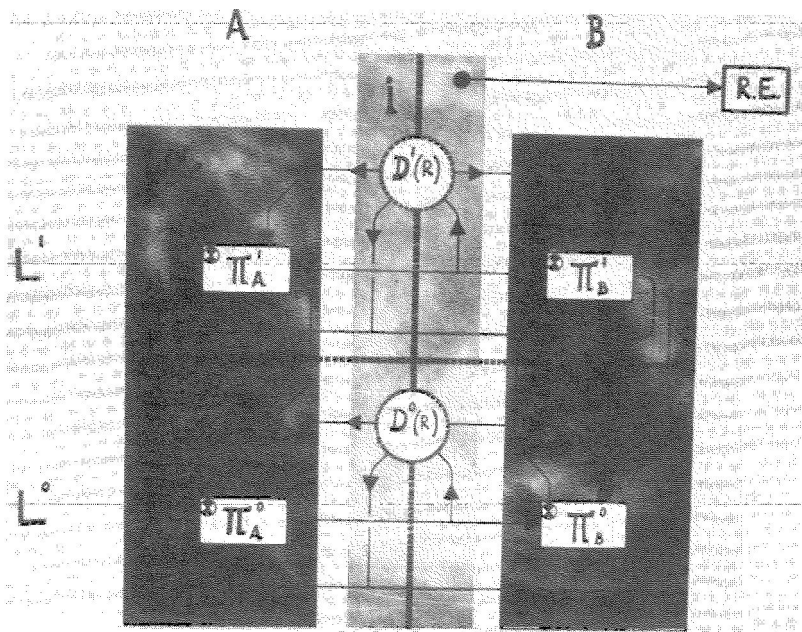


FIGURE 31. Strict conversation without modeling facility.

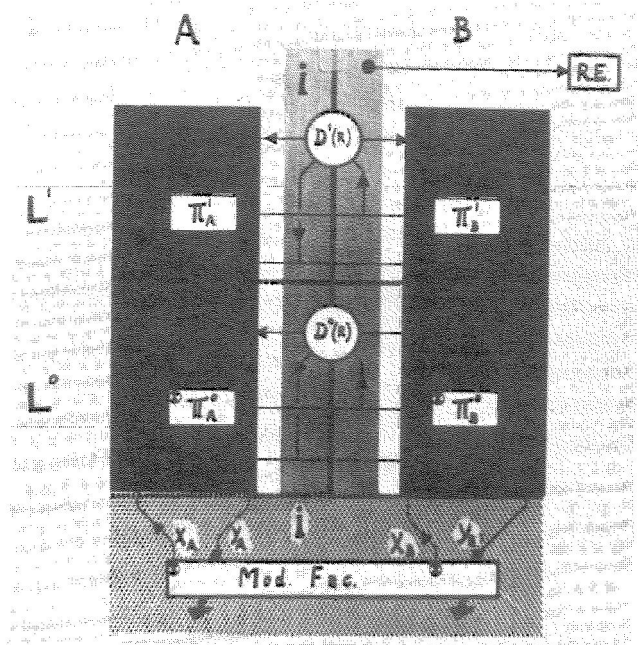


FIGURE 32. Strict conversation and specific modeling facility.

the context of B , R is not self-reproducing in the context of R . If so, B is called the support of the conversation and is denoted $S_B = \langle S_B^1, S_B^0 \rangle$ whereas participant A is ascribed the initial notation $\Pi_A = \langle \Pi_A^1, \Pi_A^0 \rangle$.

In figure 33 the P Individuals $\Pi_A = \Pi_A^1, \Pi_A^0$ and $\Pi_B = \Pi_B^1, \Pi_B^0$ have both been replaced by supports S_A and S_B . To satisfy the requirement that any L conversation is itself a P -Individual it is necessary to establish the further connections shown in this ikon (X_A to U_B , Y_A to V_B , X_B to U_A , Y_B to V_A). These constitute a different level of discourse in L , and since it is natural to think of them as discourse at a "higher" level, they are bundled together as " L^2 ." But this label is assigned on intuitive grounds, the others are not.

In figure 34 one participant (B) is a support. The other participant (A) may be a P -Individual and is labeled Π_A . If A is a P -Individual, the L^2 connection is unnecessary. Also notice that a support may be embodied in an interface although, in this construction, it is embodied in a compatible L -processor β .

The ikon in figure 35 shows the embodiment of CASTE programmed so that B is a support to A thus acting as a CET heuristic to A in the conversational domain of any fixed subject matter $D(R) = \langle D^1(R), D^0(R) \rangle$.

Figure 36 extends the ikon to include a modeling facility and its connections; for example STATLAB. Thus figure 36 is an ikon for the embodiment (or working realization) of the cognitive reflector.

The connection across the central cleft is in register with the interface, ι , so that A , B , dialogue can be recorded, understanding recognized, and these occasions in a strict conversation demarcated. Further, this conversation is anchored on the domain R .

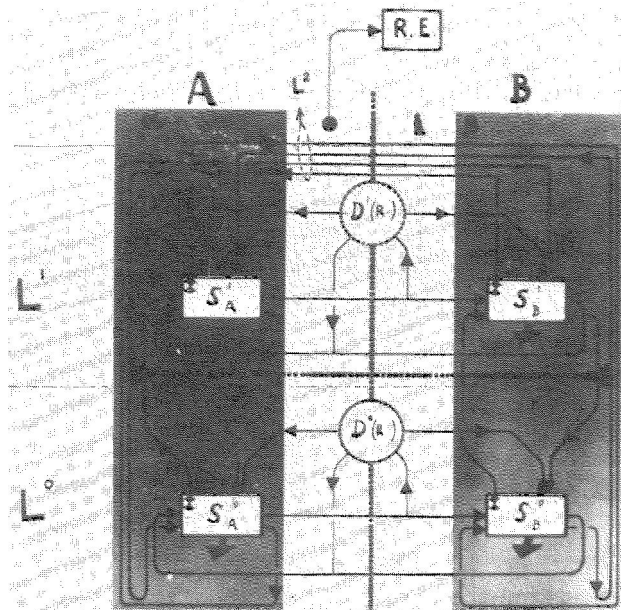


FIGURE 33.

Equivalently the (otherwise asynchronous) participants A, B are partially synchronized because of an information transfer symptomized by the dialogue.

Consider participants A, B , in strict conversation about a conversational domain R and over occasions $0, 1, \dots, n, n+1, \dots, N$ on a domain R .

If there is understanding at the sprout of this conversation upon occasion n then $n \rightarrow n+1$.

At occasion n , and subject to a caveat introduced below, let Π_A^0 stand for A 's repertoire of L^0 concepts ($\text{Proc}_A^0 i$) up to the n th occasion as well as certain primitive operations, Prim^0 , deemed common to any conceptual repertoire, and needed to construct models or explanations in the first place (Match is one, and perhaps the only, Prim^0). For any R_i in R that is ostended to anchor the sprout, $\Pi_A^0(R_i) = \Pi_A^0 i$ and $\Pi_A^0 i$ is in Π_A^0 . This statement is to be interpreted "If Π_A^0 operates on R_i at occasion n , then at or before occasion $n+1$, the Stable product $\Pi_A^0 i$, is in Π_A^0 ."

At occasion n and again subject to the caveat below, let $\Pi_A^1(n)$ stand for A 's repertoire of L^1 concepts (alias memories $\text{Proc}_A^1 i$) up to the n th occasion, as well as certain primitive operations, Prim^1 , deemed common to any cognitive repertoire, and needed to construct

The Prim^0 are isomorphic to some or all of the Prim^1 . The Prim^1 are L^1 procedures corresponding, isomorphically, to the relational operators which are the external observer's L , his metalanguage descriptions of operations that carry one or more R_i into some other R_j in R .

The promised caveat is as follows:

- (a) Call the P/P incompatibility inter-level P/P incompatibility.
- (b) Generalize the notion, to encompass intra-level P/P incompatibility, between procedures at level 0 or procedures at level 1, which, in psychological terms is interference.
- (c) No $\text{Proc}_A^0 i$ added to Π_A^0 is intralevel P/P incompatible with an existing L^0 procedure.
- (d) No $\text{Proc}_A^1 i$ added to Π_A^1 is intralevel P/P incompatible with an existing L^1 procedure.

To secure this "no destructive interference" condition the conversational domain is structured and described to satisfy the constraints already imposed upon $D(R) = \langle D^1(R), D^0(R) \rangle$ and B is designed to secure the integrity of A , which may either be stated as "continued understanding of topics in R " or as "the existence of a P Individual Π_A in the context of S_B and R ."

If B is a support (as it is in figure 34) then for any strict conversation over a domain R and occasions $0, 1, \dots, n, n+1, \dots, N$.

$$\text{Prim}^1 \subset \Pi_A^1(0) \subset \Pi_A^1(n) \subset \Pi_A^1(n+1) \subset \Pi_A^1(N)$$

and

$$\text{Prim}^0 \subset \Pi_A^0(0) \subset \Pi_A^0(n) \subset \Pi_A^0(n+1) \subset \Pi_A^0(N).$$

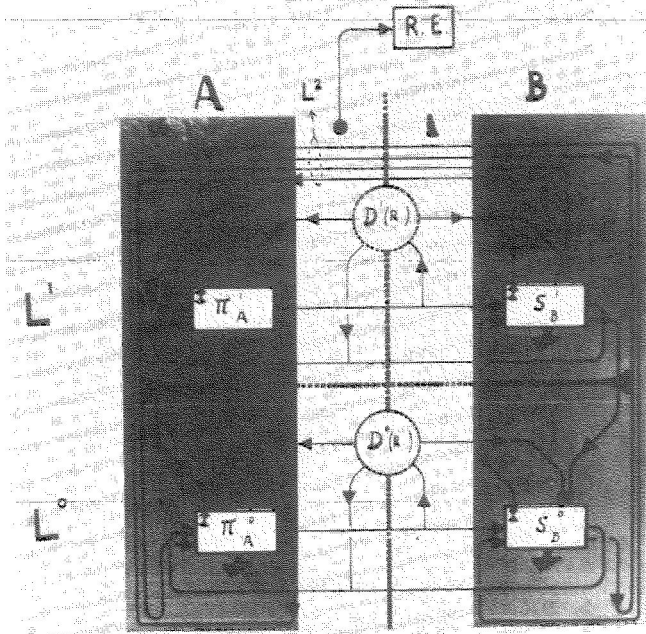


FIGURE 34. One "participant" reduced to a "support."

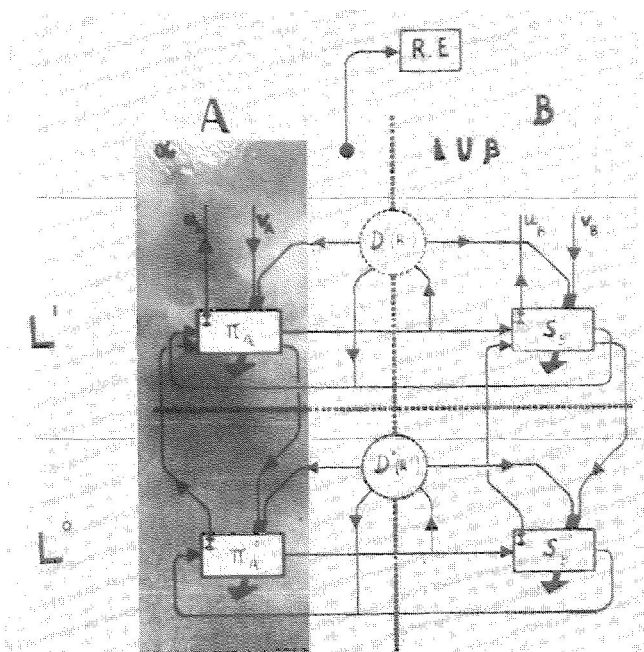


FIGURE 35. The teachback paradigm.

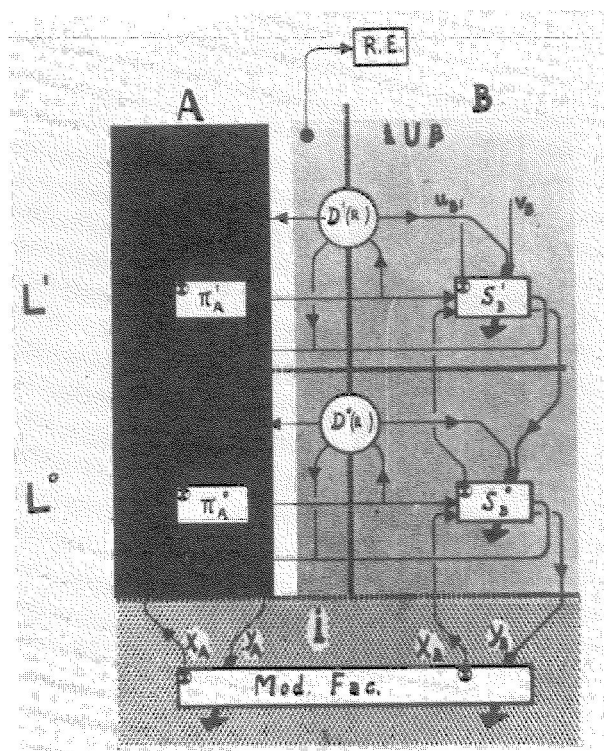


FIGURE 36. The CASTE paradigm.

This is a special case of a more general restriction, that applies to any P Individuals A, B , embodied in α, β and discussing a domain R . The restriction is manifest as cognitive fixity. A procedure, produced by learning or indoctrination upon occasion n , will be rejected from the repertoire $\Pi_A^0(n)$; it is P/P incompatible with $\Pi_A^1(n)$.

One demonstration of this property is as follows; first identify the procedures with general mappings (we may, since they are executed in specific M Individuals). Next, apply Ashby's [4] habituation theorem to show that a trapping condition exists in which the reproduction of one class of procedures inhibits the reproduction of a different class if the same P Individual is executed in the same M -Individual. Both of the underlined clauses are important; for example, cognitive fixity does not necessarily beset two or more P -Individuals under execution in the same M -Individual and it does not necessarily beset the (distributed) execution of one P -Individual over several M -Individuals. However, cognitive fixity is a property of any M localized conversation (regarded as a P Individual).

Knowledge of P/P Incompatible classes of procedures is in a state of flux. In this paper, we concentrate upon one of many possible dichotomies; namely, the incompatible classes of holist strategies and serialist strategies. Due to the mechanism cognitive fixity, these strategy classes are mutually exclusive. It is important, however, to notice the limits of the exclusion principle. The exclusion principle applies to a particular A, B , conversation on domain R . A and B might use different strategies in a different

conversational domain or conceivably on the same domain in a different conversation (or, in conversation with another participant C).

Characteristics of Standard (Classes of) Experiment in Terms of Conversations

In all cases the standard experimental situations are either more restricted than the CASTE examples or are less thoroughly controlled. There is nothing improper about this fact. Our submission in the matter is simply that fostering "good experimentation" (which, so far as man is concerned implies an "experimental contract" of some kind) is an attempt, often a successful one, to approximate the actual form of conversation to one of rather few strict forms. A sufficient number of these strict forms are shown in the following ikons, to illustrate the point.

Introspection. Figure 37 represents learning (in the present sense) taking place in one M Individual. Quite properly, Π_A is learner and Π_B his teacher, or vice versa. More generally, the ikon represents private thinking, the salient point being the coexistence of separate P Individuals, in private dialogue that is not open to conversation. One interesting category of instances are those in which Π_B depicts Π_A 's attention: another interesting category of instances are those in which Π_A and Π_B represent images of R in distinct modalities; for example, the visual and tactile representation of a rectangle.

The internal conversation is unobservable except by introspection which may impair it. Subsequently, retrospections are elicited using an interview technique to simulate a CET heuristic.

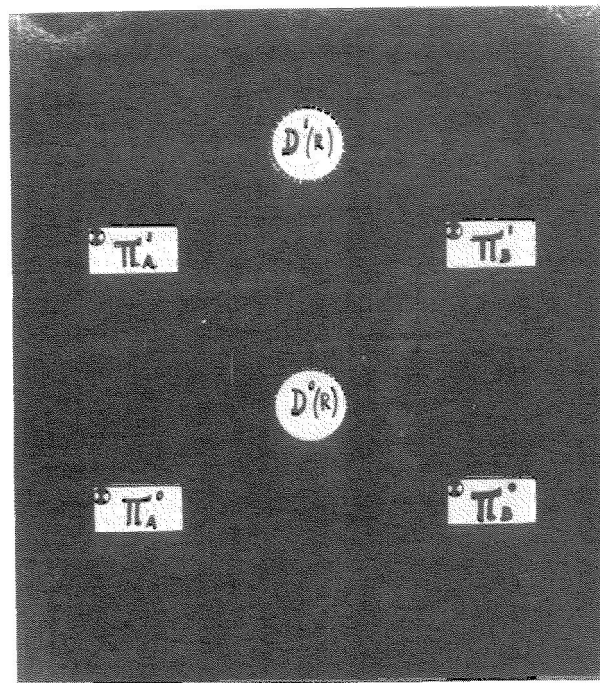


FIGURE 37. Introspection.

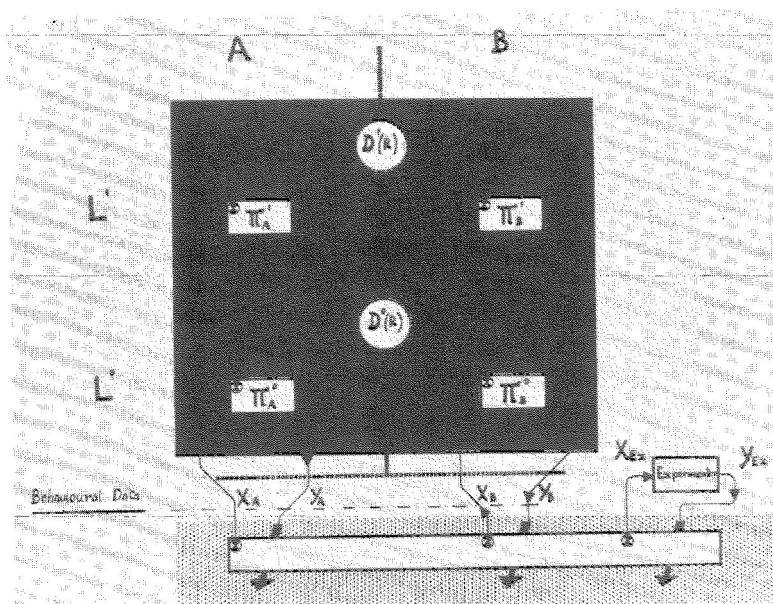


FIGURE 38. Naive interpretation of classical behavioristic experiment.

Behavioristic Experiments. Figure 38 depicts the general case for strict behavioristic observation and those kinds of cognitive experiment in which predicates are fixed, by the experimenter. By specification, the experimental interactions are causal; no participant is allowed. But a great deal can be gained by making explicit the A, B , distinction at the

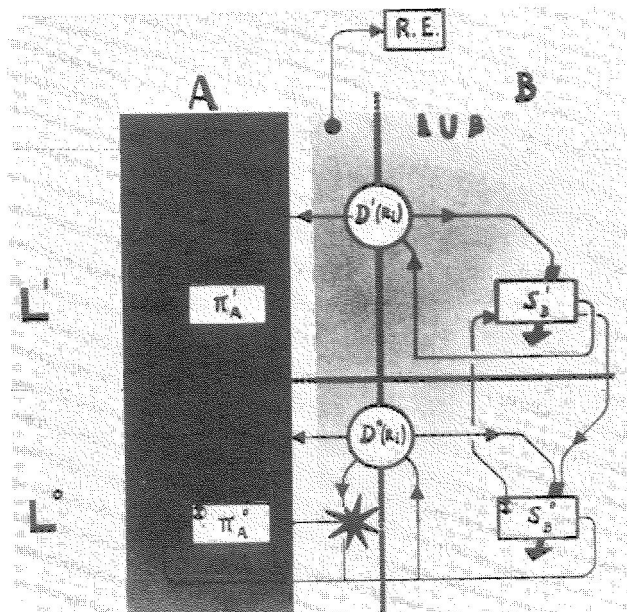


FIGURE 39. Behavioristic (in the classical sense) test and trial paradigm.

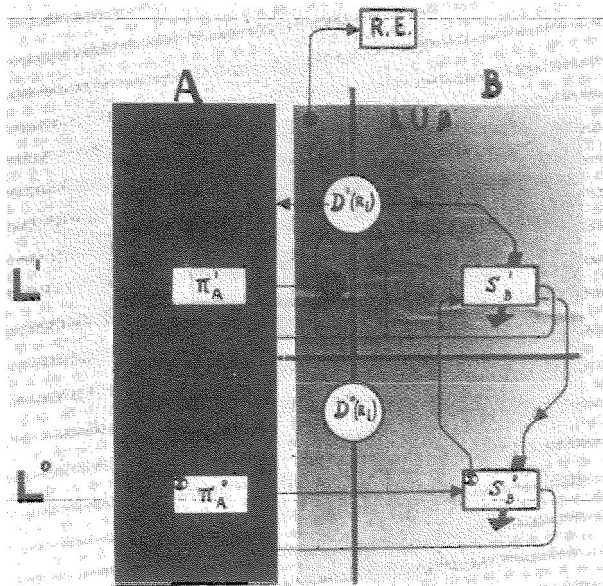


FIGURE 40. Exteriorisation of the test paradigm.

modeling facility and later scrutinizing or comparing the models built. Although we contend that the best experimental designs incorporate this expedient, some do not do so.

These comments are cogent in the field of testing and training as they are in laboratory experiments. But often mental tests employ the special notations of figure 39 and figure 40, as a specialization.

Construct Psychology. Personal constructs¹⁰ [40] are winkled out by a participant experimenter shown, henceforward, as *E*. According to the present theory, this experimenter, who may act in several modes (figures 41, 42, 43) approximates the strict conversation of an evolutionary CET heuristic.

Figure 41 shows the familiar repertory grid method. *E* presents trials of objects in a modeling facility and, at the same moment, indexes a yet to be filled in structure $D^0(R(n))$; $D^1(R(n-1))$ and $D^0(R(n-1))$ being given. The filling in, or predication, is an *L* description of a new relation (it may be a property, in the limit) as part of $D(R(n))$.

Of the methods discussed by Bannister and Mair (also in Hinkle's thesis on laddering) there are two fundamental routes for further predication. One of them (subordination) gives constructs in terms of constructs of constructs (i.e. it yields $D^0(R(n))$ by a different route (figure 42). The other method (superordination) is shown in figure 43. It yields constructs of constructs (completing $D^1(R(n))$), using existing constructs as the "objects" and differs from figure 41 insofar as we assume that the modeling facility is t clocked, and τ clocked though the interface is not.

¹⁰Our concepts are probably equivalent to constructs; our memories to constructs of constructs; are dealing either encumpanies or is equivalent to experimentations.

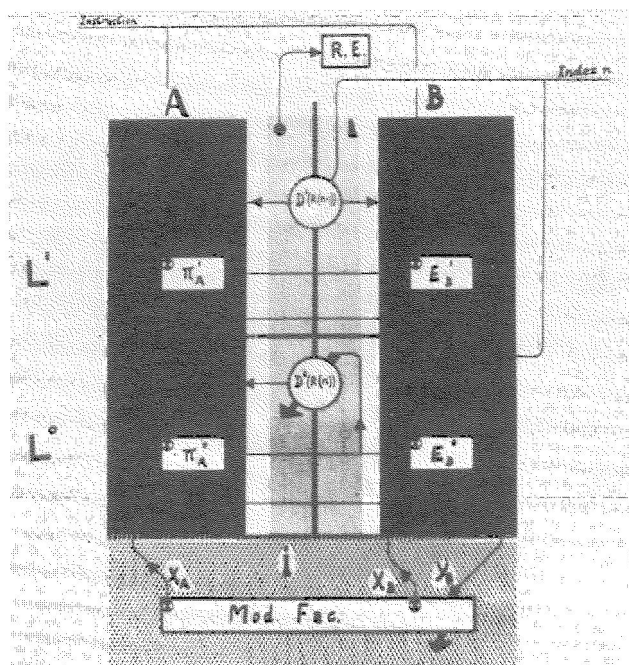


FIGURE 41. Repertory and elicitation in personal construct psychology.

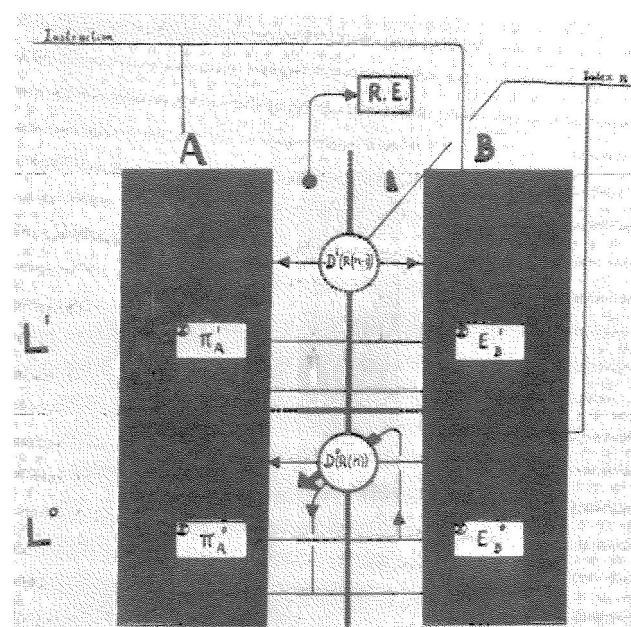


FIGURE 42. Subordination as form of repertory and technique.

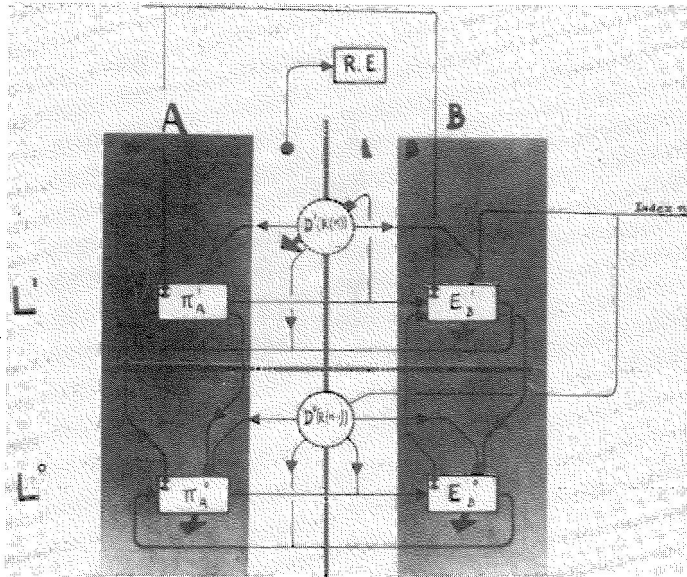


FIGURE 43. Superordination technique.

Games, Control and Problem Solving Experiments

Banerji [6] has recently described a mathematical framework able to accommodate (as special cases) game playing, control and problem solving. Since a game is the most general and often used of these situations, it is employed in developing ikons. As a sketch of Banerji's theory, the participants A and B in a game formulate strategies (albeit one move strategies) which are finite state machines and thus models in the modeling facility. A and B do so in the light of an outcome set and a payoff function $D^0(R_i)$ (where i is the experimenter's game index), and in view of a fixed norm (replacing $D^1(R)$) about how learning should take place; usually expressed as an hypothesis about A by B or vice versa (for example, the norm of rationality or of avarice).

(a) Figure 44 depicts the independent construction of an A strategy and of a B strategy, both finite state machines. Since modeling takes place $t > 0$ but $\tau = 0$.

(b) Figure 45 shows a play of the game, A 's strategy and B 's strategy are executed, usually in turns, and an outcome is obtained and evaluated by A and B namely X_A and X_B .

(c) The conduct of a competitive game consists in the arbitrarily long repetition of these ikons one after another. Index i is changed only if the game/outcome/payoff is changed and the values of i may be conceived as a series of playing instructions.

(d) For a cooperative or partially cooperative game, the players are permitted several kinds of preplay or interplay communication (Luce and Raiffa) or even bargaining under arbitration [12]. This is shown in figure 46. But the norm is still fixed and so, for any play, is R_i . However, the playing cycle becomes

→ Figure 44 → Figure 45 → Figure 46 ←

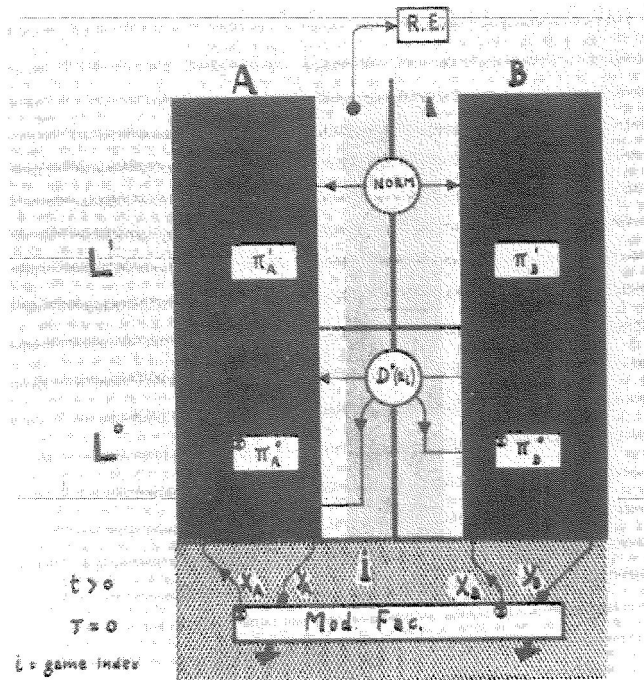


FIGURE 44. A Banerji game.

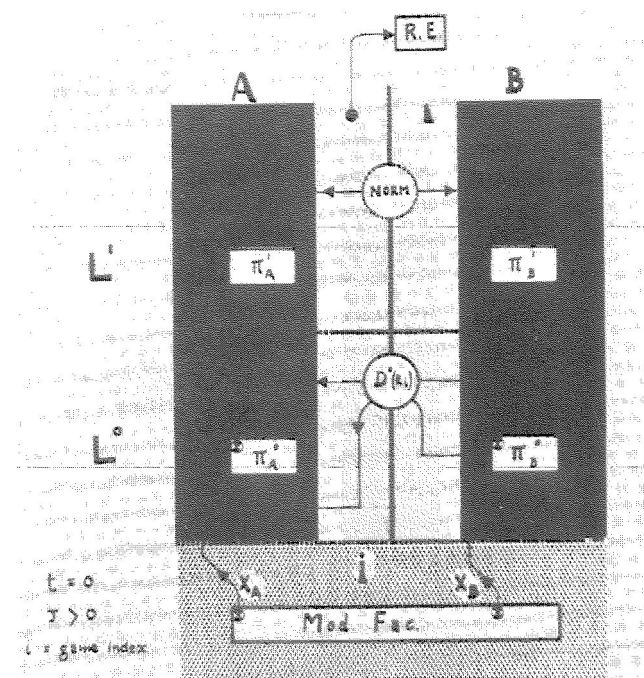


FIGURE 45. Play of a Banerji game.

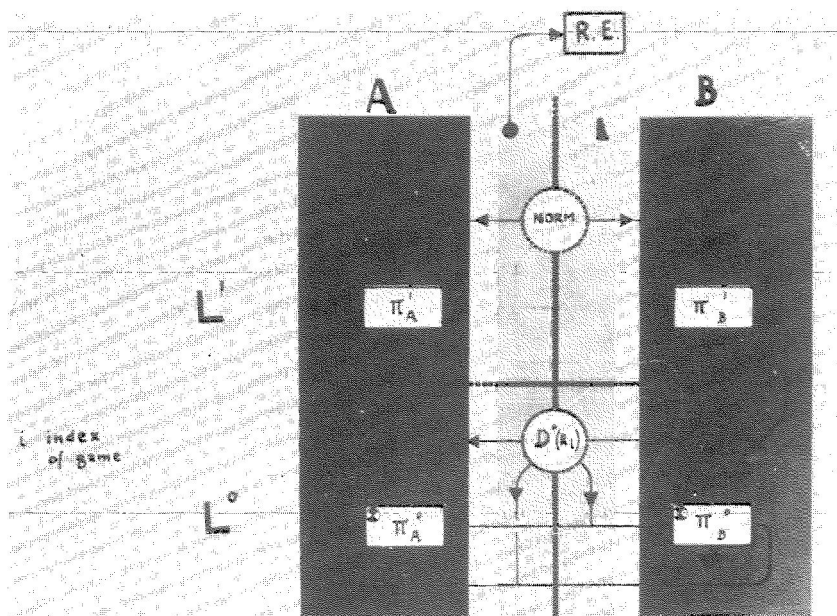
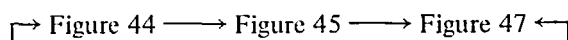


FIGURE 46. Cooperative or partially cooperative game. Play of game involves interaction and some iteration.

(e) For a metagame, in the sense of Howard [36, 37] the form of communication is distinct and the norm is allowed to change. (Player *A* can examine his moves in the light of *B*'s possible different norms by constructing normative hypotheses of his own which correspond, in extension to expansions of the outcome and payoff matrices. This kind of communication is shown in figure 47 and is bounded provided that $B^0(R_i)$ is invariant [36, 37] in the sense that further expansion reveals no further Nash solutions (i.e. dynamic equilibria in iterated play). Hence, the playing cycle becomes,



(f) These ikons are readily specialized to depict control and/or problem solving in metasystems (Pask et al., 1968, 1970 and 26, 27). It is only necessary to remark that a dynamic equilibrium in iterated play is precisely a stable mode of (possibly fuzzy) execution of the process depicted by the ikons.

Interpersonal Psychology

The final distinctive trend in psychology is spawned (chiefly) by psychoanalytic theories of the eclectic and academic type and is applicable to social as well as individual psychology. Its tennets are phrased, most precisely, by Bateson [11], Brodey [15], Laing [42] and (with neurophysiological interpretations) by Huxley [38] and Lilly [48]. Its method has been reified by the IPM test situation [43].

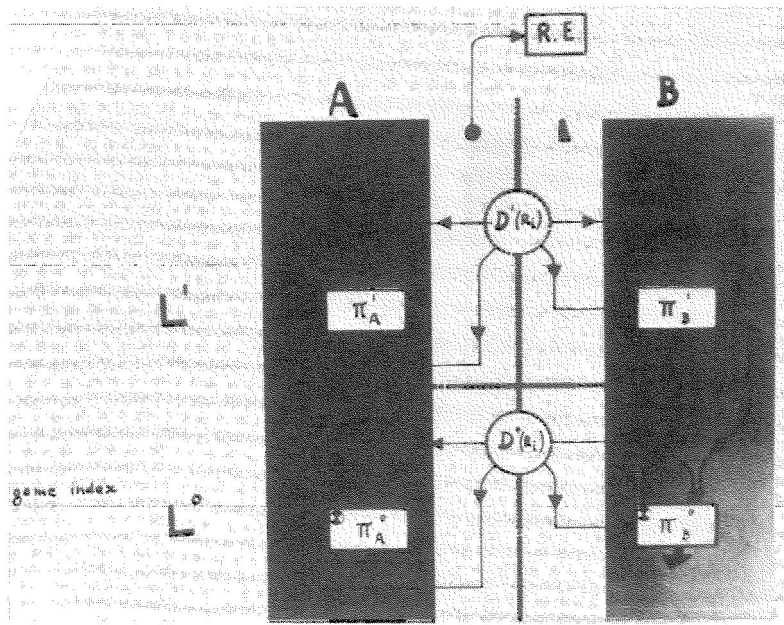


FIGURE 47. Metagame (play of metagame involves iteration).

A and B in the ikon of figure 48 are two respondents (for example, man and wife, or in our own use of the test, people in the roles of two distinctively positioned motor car drivers). A and B reply to appropriately pronominalized forms of

What do you (A) think of R_i ?

What do you (A) think B thinks of R_i ?

What do you (A) think B thinks of your (A) thinking of R_i , and of course, vice versa also in the case of B ?

The interface of figure 48 is a recording medium, but it can (as later) be much more elaborate. The experimenter delivers a sequence of indexed questions ($\text{PQuest}_A^0 i$, $\text{PQuest}_A^1 i$, $\text{PQuest}_A^2 i$, $\text{PQuest}_B^1 i$, $\text{PQuest}_B^2 i$) A 's replies and B 's replies are recorded and later discussed with the participants usually after some data processing. As noted earlier "level L^2 " is an arbitrary assignment (it might as well be called "level L^{-1} ").

Figure 49 shows a recent extension of the IPM test (a system called FRIM that delivers on line feedback during a test session with respect to the indices that Laing finds significant estimators of agreement, understanding, and appreciation). Using FRIM the test becomes a dynamic process isomorphic to the play of a metagame (a condensed shorthand for the playing cycle of Figure 44 $\rightarrow$ Figure 45 $\rightarrow$ Figure 47 and for a fixed R the truncation of levels of questioning is given by Howard's theorem).

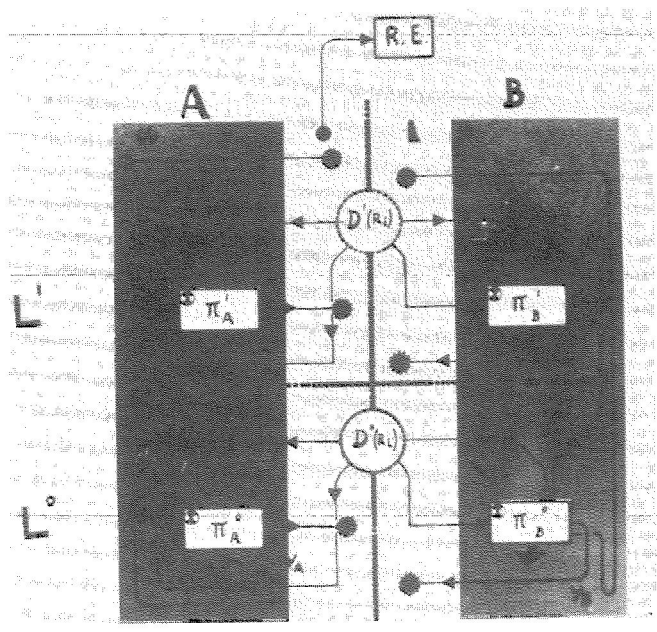


FIGURE 48. Laings IPM test.

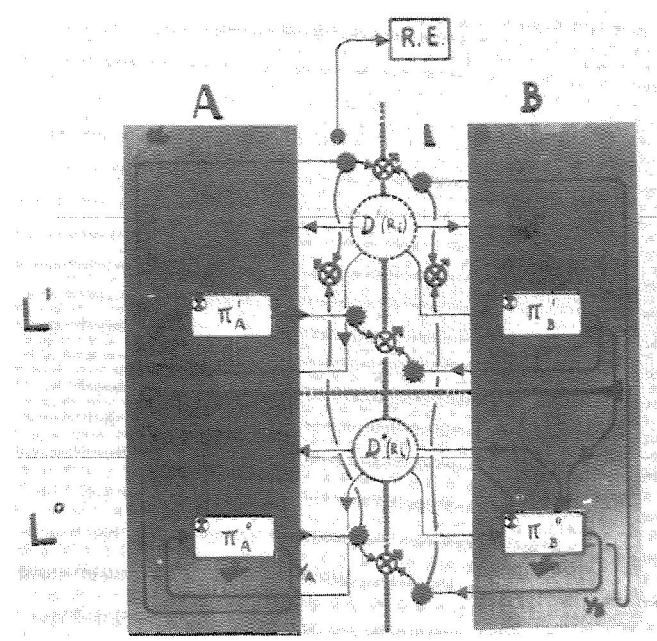


FIGURE 49. The Frim system.

Recapitulation and Generalization

The particulars can be entered into these ikons in as much detail as desired. For example, though figure 14 glosses the gamut of behavioristic experiments in what may appear to be a cavalier fashion it is possible to retrieve the lot by systematically fixing constant each parametric connection Y or description X , in turn and repeating the process for pairs etc. Hereby, the limited modes of experimentation possible under this philosophical commitment are generated. It is true that a great deal of excellent "psychological" data has, in the process, been respectfully consigned to "neurophysiology" where, according to this point of view, it belongs. Some pains might be taken to disentangle cases obfuscated in the summary literature; notably studies of "cognitive behavior" and the modern or structural "behaviorism" which is really an "information processing" psychology.

Regarding genuine cognitive studies, the present theory is at odds with none of them; neither the perspicacious eclecticism of Bartlett [10], the thoughtful functionalism of Treisman [76], Broadbent [14], and others with a psychology of doubt and uncertainty as represented by Cohen [22] or the full-blooded studies of human cognition that spring from origins in the work of James [39], Lewin [44], and Bruner, Goodnow and Austin [16]. It tallies, too, with the bases of Gestalt psychology (though obviously not with their epiphenomenal interpretations).

The theory was presented quite deliberately in the limited arena of CASTE where each facet is open to critical appraisal. Insofar as the reader is (for all that) unconvinced about the reality of predication (the point of theory we found most difficult to assimilate) the ikons of figures 41, 42 and 43 give a practical, everyday (repertory grid administration) picture of the innovative name giving that goes on in the evolutionary conversation of figure 37 whilst figures 48 and 49 do the same job for the genuine tutorial of figure 38 (which is also hard to tolerate on first acquaintance).

Perhaps the best way to come to terms with these ideas is to manipulate the ikons or, if possible, to run the corresponding programs of which listings are available on request.

The difficulties encountered in generalizing a 2-person conversation to an n person conversation ($n > 2$; a person being a P Individual the external observer chooses to distinguish) are chiefly to do with selecting an understandable method of enumeration. The exercise is mandatory if we wish (as we do) to have a social psychology and is best conceived in terms of the game ikons (figures 44, 45, 46, 47) and of their partially degenerated isomorphisms (figures 48 and 49). Since, the theory is slightly unorthodox in regard to cooperative games as part of the 2-person paradigm the problems are less obtrusive than might be anticipated. A limit is set upon game matrix expansion by Howard's theorem and is twofold for 2 participants or $n!$ fold for n participants (beyond that no further Nash solutions will be discovered or, by that token, descriptably stable notions of the system [62]. In turn this limits the maximum number of levels or horizontal clefts which must be introduced as an external observer adds further vertical clefts in distinguishing participants. For $R = R(n)$, the evolutionary case, no limit exists.

Apart from this, the following matters are outstanding and are the topics of current work. First, there is developmental psychology and the various kinds of imprinting. This field is tackled, in due course, as the *P* Individuation, by a faction in a social milieu, of a maturing and aging *M* Individual. Next, it is possible to make some cogent remarks about the psychology of extremes; notably overload on the one hand and sensory deprivation on the other. Finally, we have an ongoing research program concerned with epistemological ideas that are of primary consequence in education or, perhaps, the media of communication in general.

Appendix A

1. *P* Individuals are characterized in terms of the theory of self-reproducing automata to which, in the abstract, they correspond. This theory was developed by Von Neumann [80]; first in terms of a Turing representation and later in terms of tessellation automata. Subsequent relevant work is due to Codd and Vityani [21; 77] (for tessellation automata) to Ashby, Apter, Baricelli, Codd, Von Foerster, Weiner, McCulloch, Fogel, Owens, and Wald [1; 2-5; 21; 78; 79; 83; 53; 24] (for general and statistical models) and to Myhill and Loeftgren [56; 49] for Turing representations. Loeftgren's contribution is especially germane to the present discussion.

2. To leave the reader under no illusions, the abstract promises for my theory of conversations are chiefly culled from experience with computer simulated reproduction/evolution in systems having minimal specific structure [57; 58; 59; 60; 61]. The authors referenced have similar findings and it is fair to say that a great deal of firm knowledge exists regarding populations of reproductive automata; most of it gleaned from simulation. For example reproducibility (in a domain) specificity or compatibility, and evolution are ubiquitous.

The following comments concern the provability or otherwise of certain theorems which guarantee that such events must take place in any appropriate system.

3. With Loeftgren, we take the reproduction of a function (or, in general, a relation) Π_0 to be

$$\Pi_1(\Pi_0) = \langle \alpha, \beta, \dots \Pi_0 \rangle$$

where Π_1 may either be interpreted as an abstract machine or automaton that models Π_0 via a series of steps $(\alpha, \beta, \dots)$ or as an explanation that proves Π_0 starting with the axiom(s) α . In the theory of strict conversations Π_0 is identified with a relation in the conversational domain and Π_1 with a procedure Proc 1 which is a concept of R_0 if and only if it reproduces R_0 ; that is $\text{Proc 1}(R_0) = \langle \alpha, \beta, \dots R_0 \rangle$. The construction ending in R_0 is called a modeling operation or, synonymously, an execution sequence of operations elicited by a command. An *L* description of this sequence is written $D(\langle \alpha, \beta, \dots R_0 \rangle)$. It constitutes an explanation of R_0 and might have been elicited directly (as an *L* expression ending in $D(R_0)$) by an explanation demanding question.

4. As a matter of convenience, any reproduction of a reproductive agent is usually distinguished to avoid self reference. That is,

$$\Pi_2(\Pi_1) = \langle u, S, \dots \Pi_1 \rangle$$

when Π_1 operates on a domain containing Π_0 but not Π_1 and Π_2 operates on a domain containing Π_1 but not Π_2 . So, for example, Proc 2 (Proc 1) = $\langle u, S, \dots \text{Proc 1} \rangle$ is equivalently cited as the reproduction of a concept Proc 1 or as a memory of R_0 .

The requisite constraints are imposed by stratifying the conversational object language, L , into levels of control. This expedient is used extensively in the sequel and it is important to recognize that the strata are quite arbitrary; they do not image some property of real things. For instance, it is possible and permissible to choose rules that admit entities that are symbiotically self-reproducing, like

$$\left\{ \begin{array}{l} \Pi_1(\Pi_2) = \langle \alpha^*, \beta^*, \dots \Pi_2 \rangle \\ \Pi_2(\Pi_1) = \langle u^*, S^*, \dots \Pi_1 \rangle \end{array} \right\} \text{ Repro 1}$$

or even (Loefgren's complete self-reproducibility).

$$\{\Pi(\Pi) = \langle \eta, \mu, \dots \Pi \rangle\} \text{ Repro 2}$$

The organizations Repro 1 and Repro 2 are self-reproducing systems that image P Individuals.

The advantages and disadvantages of stratification have been discussed at length by Gorn [30]. Stratified control of one process by another pays dividends insofar as the processes in question can be unambiguously observed and described. However if the stratification is complete, then no self-referential process is allowed. In contrast, unstratified control (Gorn's name for a linguistic structure that does accommodate Repro 1 and possibly Repro 2) though nearer to characterizing natural language, does not ensure precise and unequivocal observation. The theory of strict conversations is based upon a compromise; to some extent due to a remark of Gorn's [30] that the 1966 version of the theory was largely couched in terms of unstratified control and, because of that, was beset by certain unnecessary limitations.

5. A strict conversation is a P Individual the execution of a symbiotically self-reproducing system (in the sense of Loefgren's [49] development) with respect to a conversational domain R (a surrounding in Loefgren's sense though R , in fact, may include other reproductive systems).

The requisite mathematical development is chiefly contained in one paper [49] to which the following proof numbers refer.

(a) The existence of productive and self-productive machines in a domain (alias a surrounding) R is shown by Theorem 1.5 (p. 361).

(b) The existence of a symbiotically self-reproducing system is shown by Theorem 11.5 (p. 361).

(c) Any such system has its symbions as factors in R .

(d) The minimal factor is self-productive (but not reproductive) in R .

(e) Factoring may be represented, in dynamic models (such as the ikons) by reducing each reproducible loop (one at least for each level) either into commands/executions or questions/replies.

(f) The reproductive automata that perform other operations, apart from reproduction, is given in Theorem 11.9 (p. 364).

(g) The existence of indefinitely many automata produced by one automaton in a surrounding (hence learning or evaluation) is shown by Theorem 11.10 (p. 365).

(h) The concept of *P/P* compatibility is evident from the preamble to these theories (for example, change the characterization of the surrounding). It is more specifically exhibited, in the context of genetic *P/P* compatibility by Vityani [77].

Appendix B

The logic of commands (imperative logic; part of Von Wright's [81] deontic logic) and the logic of questions (especially those that call for an explanation in reply) are quite distinct from assertoric logic. Rescher's [70] logic of commands is, so far as I know, the only detailed essay in this field and it provides a convenient starting point for the discussions.

1. Simple Command and Question

1(1). Rescher [70] notes that a command $\text{Comm}_A i$ is issued, on authority, to some addressee, A ; it is to be executed if and only if certain preconditions called *Precon* are satisfied and it either specifies an action to be performed or a further condition (Rescher's "state achievement" commands) to be brought about. In general, we are concerned with "state achievement" commands and represent them in the form

$$\text{Comm}_A i = \langle A!, R_i / \text{Precon } i \rangle$$

where A may either be one individual or a group and, in the latter case, where $\text{Comm}_A i$ can be issued collectively to every member of group A or distributively, to any one or subgroup of members of A . R_i , as usual, is a relation.

1(2). $\text{Comm}_A i$ is terminated if on some occasion n , $\text{Comm}_A i$ was issued to A either on this or on some previous occasion and if, at n , the following L^* statement is true "at n , A brought about R_i , given *Precon*" (recall that L^* is the external observer's metalanguage).

1(3). Rescher calls commands of this type (in contrast to sequences of commands having contingent connection) simple commands. For reasons that appear in a moment they are, here, regarded as Base Commands $\text{Comm}_A i = \langle A!, R_i / \text{Precon } i \rangle$ and the converse interrogative form is an explanation-demanding-question namely $\text{EQuest}_A i = \langle A? \text{ Expl } i / R_i \rangle$.

1(4). For (Base) commands $\text{Comm}_A 1$, $\text{Comm}_A 2$, it is said that $\text{Comm}_A 1$ covers $\text{Comm}_A 2$ or $\text{Comm}_A 1 \gg \text{Comm}_A 2$ if and only if $\text{Termination}(\text{Comm}_A 1) \Rightarrow \text{Termination}(\text{Comm}_A 2)$.

1(5). It may be that one command covers another only in the context of a particular L statement, adjoined to the Command and serving as part of Precon. If $\text{Comm}_A^0 i$ covers $\text{Comm}_A^0 2$ only in a certain context this fact is symbolized as (Rescher's [70] contextual coverage relation) by $\text{Comm}_A^0 1 > \text{Comm}_A^0 2$.

1(6). A (Base) command $\text{Comm}_A 0$ may be decomposed into constituents $\text{Comm}_A 1$, $\text{Comm}_A 2$ where $\text{Comm}_A 1 = \langle A_1! R_1/\text{Precon } 1 \rangle$ and $\text{Comm}_A 2 = \langle A_2! R_2/\text{Precon } 2 \rangle$ and $\text{Comm}_A 0 = \langle A! R_0 \text{Precon} \rangle$ if either

(a) $A_1 \cup A_2 = A$ and $R_1 = R_2$ and $\text{Precon } 1 \equiv \text{Precon } 2 \equiv \text{Precon}$; or

(b) $A_1 = A_2 = A$ and (if both R_1, R_2 hold, then, R holds) and $\text{Precon}^1 \equiv \text{Precon}^2 \equiv \text{Precon}$; or

(c) $A_1 = A_2 = A$ and $R_1 = R_2 = R$ and $(\text{Precon } 1 \vee \text{Precon } 2) \supset \text{Precon}$

A decomposition of $\text{Comm}_A 0$ is a list of commands derived whilst obeying these rules.

1(7). The termination of $\text{Comm}_A 0$ implies the termination of all commands in any of its decompositions.

1(8). The logical connectives corresponding to “ $\wedge$,” and the “or” of assertoric logic are fusion and disjunction. They are specified for commands issued to the same addressee and (in the case of disjunction) having the same precondition.

(a) $\text{Comm}_A 0$ is the fusion of $\text{Comm}_A 1 = \langle A! R_1/\text{Precon } 1 \rangle$ and of $\langle \text{Comm}_A 2 = \langle A! R_2/\text{Precon } 2 \rangle$; written, “ $\text{Comm}_A 1 \otimes \text{Comm}_A 2$ ” or “ $\otimes$ ” ($\text{Comm}_A 1, \text{Comm}_A 2$) if and only if $\text{Comm}_A 0 = \langle A! (\text{bring about both } R_1 \text{ and } R_2)/\text{Precon } 1 \cup \text{Precon } 2 \rangle$.

(b) Two forms of disjunction $\text{Comm}_A 1 \vee \text{Comm}_A 2$ and $\text{Comm}_A 1 \bar{\vee} \text{Comm}_A 2$ are considered by Rescher [70] (since he conceives the command, qua program, as undergoing serial execution he does not examine others). Of these $\text{Comm}_A 1 \vee \text{Comm}_A 2$ means “either $\text{Comm}_A 1$ or (exclusively) $\text{Comm}_A 2$ whichever you choose” (in this paper choice is the truncation of a process as shown in figure 48 and figure 49). The other form $\text{Comm}_A 1 \bar{\vee} \text{Comm}_A 2$ means “either $\text{Comm}_A 1$ or (exclusively) $\text{Comm}_A 2$ depending upon circumstances,” which is interpreted as “depending upon a condition holding,” or depending upon “a modification or qualification of the precondition.” Generally, a condition is not a state, and if the precondition is a state it may be modified by an L statement. Termination of $\text{Comm}_A 1 \vee \text{Comm}_A 2$ or of $\text{Comm}_A 1 \bar{\vee} \text{Comm}_A 2$ implies the termination of $\text{Comm}_A 1$ or of $\text{Comm}_A 2$ or both.

1(9). To derive a further meaning for disjunction (more like inclusive disjunction) we need Rescher's [70] concept of command preemption and preclusion (wedded in an appropriate fashion, to our ideas of instant and occasion).

1(10). At an instant t_n , in occasion n , $\text{Comm}_A 1$ preempts/precludes $\text{Comm}_A 2$, if, at t_n , termination ($\text{Comm}_A 1$) is inconsistent with termination ($\text{Comm}_A 2$). The preemption/preclusion applies at t_n , the instants of execution (and could be due either to logical or local considerations; for example, that +1 and -1 appear simultaneously in one register). The general case is a set of μ commands $\text{Comm}_A \eta$; $\eta = 1, \dots, \mu$ so coupled in execution that any preemption/preclusion is prohibited (“no conflict,” a condition that may depend on the processor as well as the process). Under these circumstances a command to execute all μ of the $\text{Comm}_A \eta$ is written $\text{Comm}_A 1 \check{\vee} \dots \check{\vee} \text{Comm}_A \mu$, or $\check{\vee}_{1,\mu}$ ($\text{Comm}_A \eta$; an “all but precluded” disjunction).

1(11). These comments sketch out Rescher's [70] argument, though the terminology (but not the meaning) has been modified. For example, in choosing a P Individual A as the addressee (rather than an M Individual α, β) we buttress Rescher's comment that a command is an L program that is addressed to an executive agent able to interpret L . In the original, there is no distinction between the interpretative and executive organization and the processor (i.e. A and α, B and β , are one to one identified). Here, we explicitly address the command (or its interrogative form) to a processor independent organization able to interpret L by using L statements like $\text{Comm}_A^0, \text{Comm}_B^0$ or $\text{EQuest}_A^0, \text{EQuest}_B^0$. To do so is not at odds with the original; to issue a command to α or β would be nonsensical except in the case when α executes A or β executes B .

The twist in the presentation is that a termination statement is an L^* statement, not an L statement. Any termination statement refers to the agent and adjudicates his (or its) action; the truth or falsity of this statement is determined in a metalanguage for talking about L processes and their execution. In contrast, of course (by including an innate match operation) it is possible for A and B to agree in L (not in L^*) that they have executed the same command (i.e. $\text{Exec}_A^0 i \Leftrightarrow \text{Exec}_B^0 i$) or given the same explanation ($\text{Expl}_A^0 i \Leftrightarrow \text{Expl}_B^0 i$) moreover, it is up to an external observer to design B so that A, B agreement means (at the metalevel of L^*) termination. This trick is used in specifying B so that A, B agreement means " A has made a correct model" or " A has given a correct explanation."

The only departure from the original is the introduction of $\checkmark$ disjunction in 1(10) and this, as below, appears to be a legitimate generalization of Rescher's point of view.

There is no difficulty in giving the same command to a group of addressees to whom it may be addressed either in a distributive or collective manner (as in 1(1)). For example, $\text{Comm}_A^0 i, \text{Comm}_B^0 i$ is a command addressed to A and B as a group (namely the A, B conversation) to bring about R_i given Precon i . It either means that both of A, B should bring about R_i (the collective sense) or one of them should do so (the distributive sense). The joint injunction is not trivial, even though A and B are commanded to do the same thing, insofar as they have distinct Proc^0 s; that is $\text{Proc}_A^0 l, i \neq \text{Proc}_B^0 l, i$ though both $\text{Proc}_A^0 l, i$ and $\text{Proc}_B^0 l, i$ bring about R_i if Precon is given.

We interpret any such a command in its distributive form as a $\vee$ or $\bar{\vee}$ disjunct.

"Either A (exclusively) bring about R_i or B (exclusively) bring about R_i " or, in its collective form, as the $\checkmark$.

"Both A and B bring about R_i by your own (compatible) methods."

Thus $\checkmark$ calls for the parallel and independent execution of $\text{Proc}_A^0 l, i$ and $\text{Proc}_B^0 l, i$. No issue of compatibility arises provided A and B act in isolation until the (same) result, R_i is achieved redundantly.

The case when A and B are executed in the same processor (α say) poses no fresh problem provided that isolation is maintained. As a straightforward generalization A may consist in μ distinct processors A_η : $\eta = 1, \dots, \mu$; at least one of which has $\text{Proc}_A^0 \eta, i$ for bringing about R_i though there is no requirement that $\text{Proc}_A^0 \eta, i$ (in A_η) is the same as $\text{Proc}_A^0 \eta+1, i$ (in $A_{\eta+1}$) and, in interesting cases, it will not be.

If the A_η are independent (as assumed), then no incompatibility is possible and a command that all of the μ separate A_η should independently (but perhaps

simultaneously) operate to bring about the same R_i using different methods is again interpreted as the collective injunction. Thus, “ $\langle(\text{All } A_\eta \text{ in } A!) \text{ bring about } R_i/\text{Precon}\rangle$ ” may be rephrased as the disjunctive form.

$$“\checkmark \langle A! \text{ bring about } R_i \text{ using } \text{Proc}_A^0 \eta, i / \text{Precon} \rangle.”$$

1(12). The reader will rightly sense that the A_η are to feature in the sequel as stable concepts¹¹ (alias procedures) in A 's repertoire Π_A^0 ; any $\text{Proc}_A^0 \eta, i$ being characterized as a distinct method for bringing about R_i because $\text{Proc}_A^0 \eta, i$ is an arrangement of different operations from those employed in (say) $\text{Proc}_A^0 \eta + l, i$ or because it is a different arrangement of the same operations. Under these conditions the A_η (that is, the stable $\text{Proc}_A^0 \eta, i$) are no longer independent. They are prone to be interact at intermediary stages in computation and some of the interactions may be contradictory. If the independence clause is dropped, then the $\checkmark$ disjunct evokes concurrent rather than parallel (and independent) redundant computation. Preclusion/preemption was introduced with the $\checkmark$ disjunct in 1(10) to obtain a “no conflict” condition under which concurrent computation would be stripped of contradictory (i.e. precluded/preempted) subconditions at stages when only some of the operations have been performed. We are about to give the base command $\text{Comm}_A^0 i$ one interpretation as a $\checkmark$ disjunct. Requiring A to bring about R_i by all legal methods at his disposal where “legal” means “represented in $D^0(R_i)$.”

2. Learning and Doing

2(1). Base Commands $\text{Comm}_A i$ (and the corresponding interrogations) can be issued and obeyed at either of the levels in L ; namely L^1 or L^0 .

Thus an L^1 command as an instruction to build a $\text{Proc}_A^0 i$ and an L^0 command is an instruction to bring about R , using one or more of the $\text{Proc}_A^0 l, i$ in Π_A^0 .

2(2). Diagram I is an indexing key showing two fragments of $D^0(R)$, a conjunctive substructure and a disjunctive substructure. It is used as a prop for the following discussion of command forms and interpretations.

2(3). For the conjunctive substructure in Diagram I and l an index over distinct procedures in $\Pi_A^0(n)$ at occasion n

$$\text{Comm}_A^1 i = \langle A! \text{ Proc}_A^0 l, i / \text{Proc}_A^0 l, p \text{ in } \Pi_A^0(n), \text{ and } \text{Proc}_A^0 l, q \text{ in } \Pi_A^0(n), \text{ and } \text{Prim}_A^0 F \text{ in } \Pi_A^0(n) \rangle.$$

The precondition

$$\text{Proc}_A^0 l, p \text{ in } \Pi_A^0(n) \text{ and } \text{Proc}_A^0 l, q \text{ in } \Pi_A^0(n) \text{ and } \text{Prim}_A^0 F \text{ in } \Pi_A^0(n)$$

is unambiguous and unique.

¹¹Stable in the sense that for any $\text{Proc}_A^0 \eta$; there is some Proc_A^1 ; that reproduces η as a memory.

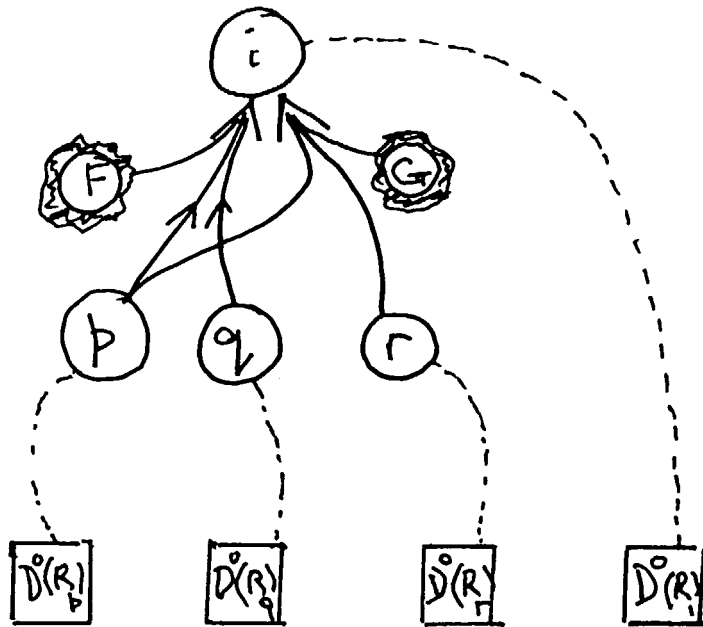
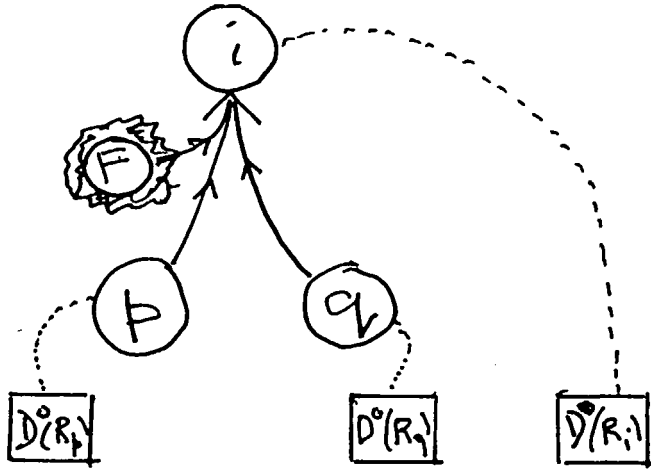


DIAGRAM I. A fully conjunctive substructure in $D^1(R)$ and a disjunctive substructure in $D^1(R)$ together with the $D^0(R_i)$ that are attached to them by data links (shown as dotted lines).

2(4). For the disjunctive substructure in Diagram I, the L^1 base command has two preconditions, namely

$$\text{Precon } 1, i \equiv \text{Proc}_A^0 l, p \text{ in } \Pi_A^0(n) \text{ and } \text{Proc}_A^0 l, q \text{ in } \Pi_A^0(n) \text{ and } \text{Prim}_A^0 F \text{ in } \Pi_A^0(n)$$

and

$\text{Precon } 2, i \equiv \text{Proc}_A^0 l, p$ in $\Pi_A^0(n)$ and $\text{Proc}_A^0 l, r$ in $\Pi_A^0(n)$ and $\text{Prim}_A^0 G$ in $\Pi_A^0(n)$

Hence, there are really two L^1 base commands, namely

$$\text{Comm}_A^1 1, i = \langle A! \text{Proc}_A^0 l, i / \text{Precon } 1, i \rangle$$

$$\text{Comm}_A^1 2, i = \langle A! \text{Proc}_A^0 l, i / \text{Precon } 2, i \rangle$$

2(5). Suppose that both preconditions are satisfied, at an occasion n , then $\text{Comm}_A^0 i$ (actually having the two conditional forms just exhibited) may be given two distinct interpretations; a weakest or C^- interpretation and a strongest or C^+ interpretation. These correspond to the command disjunctions.

Interpretation C^- if $\text{Comm}_A^0 i$ is $\forall (\text{Comm}_A^0 1, i; \text{Comm}_A^0 2, i)$

Interpretation C^+ if $\text{Comm}_A^0 i$ is $\check{\forall} (\text{Comm}_A^0 1, i; \text{Comm}_A^0 2, i)$

2(6). The construction of $D^1(R)$ guarantees that if A receives $\text{Comm}_A^1 i$; at occasion n when at least one of the preconditions is satisfied (perhaps both) then there is no preclusion which prohibits giving the $\check{\forall}$ interpretation to $\text{Comm}_A^0 i$. Of course, the $\text{Proc}_A^0 l, i$ built by A are almost certain to differ according to which interpretation A does give to $\text{Comm}_A^1 i$. But the isomorphism asserted in $D^1(R)$; namely $R_i \Leftrightarrow (F(R_p, R_q)) \Leftrightarrow (G(R_p, R_r))$, guarantees that whatever $\text{Proc}_A^0 l, i$ is (are) built will serve to provide the cyclicity or reproducibility conditions for all $\text{Comm}_A^1 \xi$ such that its Precon contains a term “ $\text{Proc}_A^0 l, i$ in $\Pi_A^0(n + m)$ ”; $m \geq 1$ where node ξ is superordinate to node i in $D^1(R)$.

2(7). In other words R_i is functionally the “same” relation in the context of $D^1(R)$ however it is learned and subsequently satisfied.

For example, suppose A is learning shopkeeper’s accountancy (a relation R_{head} in $D^1(R)$) and is required to perform the usual arithmetic operations in order to exercise that cluster of skills, so R_i is, for example, division of real numbers.

A might have learned division in many different ways (since node i surmounts a disjunctive substructure in $D^1(R)$), A may, as a result, be able to divide by one or many different methods (for example by long division, by the “Chinese method” etc.). But, so far as the condition of cyclicity and subsequent reproducibility is concerned, it does not matter in what way he learned division and further, all methods of division are said to be equivalent in the context of accountancy skills as their knowable content is represented in $D^1(R)$.

2(8). It should be emphasized that these statements are very specific. It is neither asserted nor believed that A ’s interpretation of $\text{Comm}_A^1 i$ (as $\forall$ or $\check{\forall}$) is immaterial in

general, or that it has no influence on A 's psyche or learning rate, or his immunity, in contexts other than $D^1(R)$ to operations that interfere with division.

On the contrary, all of these general properties are influenced if A is required to interpret L^1 commands in a particular manner. But the word "influence" is used guardedly; for, although it is possible to insist upon a $\check{V}$ interpretation if otherwise a $\check{V}$ interpretation might have been given there is no obvious way of insisting that A chooses one learning method ($\check{V}$) if A opts for $\check{V}$ and $D^1(R)$ is disjunctive.

2(9). An L^0 base command $\text{Comm}_A^0 i$ is a directive that A should make a model that brings about R_i within the constraints imposed by a modeling facility.

2(10). The precondition of $\text{Comm}_A^0 i$ (Precon i) is the occasion n when $\text{Comm}_A^0 i$ is issued together (for all nodes j in at least one kernel of R_i), with the names of $\text{Proc}_A^0 j$ in $\Pi_A^0(n)$ and the values they compute. In other words Precon i is n together with the coordinates of R_i that are subordinate relations which A , at occasion n , is able to regard as properties. These properties (the values of which are related by R_i) are used as predicates in A 's description of his universe of modeling; the state set $X_A \bar{\subset} X$.¹²

2(11). $\text{Comm}_A^0 i$ is usefully compared and contrasted with $\text{Comm}_A^1 i$ (2(1). to 2(6).) as follows:

2(12). $\text{Comm}_A^1 i$ has a precondition that is an occasion n together with the existence in $\Pi_A^0(n)$ of $\text{Proc}_A^0 j$ which compute (for each node j in at least one kernel of node i) the subordinate relations R_j . The command is a directive that A should use these procedures in order to construct a further procedure $\text{Proc}_A^0 l, i$.

2(13). $\text{Comm}_A^0 i$ has a precondition that is an occasion n together with names of the $\text{Proc}_A^0 j$ and the values they compute. The command itself is to execute $\text{Proc}_A^0 i$ to produce $\text{Exec}_A^0 i$ (a model in the modeling facility that preserves R_i) or $\text{Expl}_A^0 i$ (an explanation which starts with any values of the coordinates of R_i and shows how they are related by R_i).

2(14). For the conjunctive substructure under node i , L^0 base commands to bring about relations tagged by nodes subordinate to R_i belong to a decomposition of $\text{Comm}_A^0 i$. Thus they are strictly covered by $\text{Comm}_A^0 i$. Thus in diagram I

$$\text{Comm}_A^0 i \gg \text{Comm}_A^0 p$$

$$\text{Comm}_A^0 i \gg \text{Comm}_A^0 q$$

2(15). In particular, these commands refer to the same modeling facility, and are restrictions of the same universe.

$$\text{Precon } i \Rightarrow (\text{Precon } p \wedge \text{Precon } q)$$

2(16). The same comments apply to any one kernel of a disjunctive node but not, as a rule, to all kernels. So, in the disjunctive substructure of diagram I.

¹²Phrased differently X is a set of storage positions to hold the values of yet to be declared variables. X_A is the set of conjoint values of the variables that are declared by A and that are stored in these positions.

$$\text{Comm}_A^0 i \gg \text{Comm}_A^0 p$$

and either

$$\text{Comm}_A^0 i \gg \text{Comm}_A^0 q \text{ or } \text{Comm}_A^0 i \gg \text{Comm}_A^0 r \text{ or both}$$

2(17). Further $\text{Precon } i \Rightarrow (\text{Precon } p) \vee (\text{Precon } q \vee \text{Precon } r)$.

In general, there is more than one modeling facility and the modeling facilities may only be related through $D^1(R)$ (the matter is taken up again in 4).

2(18). If A engages in a strict conversation regulated by a CET heuristic there is in addition a well defined contextual coverage (of 1(5).) for each occasion n .

In the context of A 's learning strategy up to occasion n (an L statement, made explicit in 5(20).) $\text{Comm}_A^0 i >$ any node marked understood.

3. Command Sequences, Commands, Graphs, and $D^0(R_i)$

3(1). $D^0(R_i)$ is a description of what may be done to bring about topic relation R_i within the constraints imposed by a modeling facility; with more generality $D^0(R_i)$ is a description of those $\text{Proc}_A^0 l, i$ (l an index over different procedures) that may legally exist as concepts in Π_A^0 .

3(2). Let C be a command form or program with a variable representing its addressee. Thus

$$C_i = \langle Z! R_i / \text{Precon } i \rangle = \begin{cases} \text{Comm}_A^0 i, & \text{if } Z = A \\ \text{Comm}_B^0 i, & \text{if } Z = B, \text{ and so on} \end{cases}$$

3(3). A particular command form, C_i is a command form sequence, namely $SC\ l, i = \langle C_d^{\blacksquare}, C_e^{\blacksquare} \dots \rangle$ where the $C_d^{\blacksquare}, C_e^{\blacksquare}$ are subprograms; either Prim^0 or $\text{Proc}_A^0 i$ in Π_A^0 and the execution of $SC\ l, i$ leads to R_i .

On expansion, a two term sequence (for example) becomes

$$\begin{aligned} C_{l,i} = \langle C_1^{\blacksquare}, C_2^{\blacksquare} \rangle &= \langle \langle Z! R_i / \text{Precon } 1 \rangle, \langle Z! R_2 / \text{Precon } 2 \rangle \rangle \\ &= \langle \langle Z! R_i / \text{Termination } C_2^{\blacksquare} \rangle, \langle Z! R_2 / \text{Precon} \rangle \rangle \end{aligned}$$

which, in Rescher's [70] exposition is a chain of commands (when Z is given a definite value as it must be if $C_{l,i}$ is issued, for example, to $Z = A$ or $Z = B$).

3(4). Let $\{SC\}_i$ be a set of $SC\ l, i$: in which the different command form sequences all bring about R_i if executed under Precon.

$\{SC\}_i$ is a limiting case of a $D^0(R_i)$. It describes all command chains involving only Proc_A^0 in Π_A^0 or Prim^0 in Π_A^0 that might be issued (by B , for example, to A) each of which, if executed, secure R_i .

3(5). Since an $SC\ l,i$ is a specific method or program for bringing about R_i it covers C_i . Thus $SC\ l,i \geq C_i$ for all $SC\ l,i$ in $\{SC\}_i$.

3(6). It follows that if adherence to $D^0(R_i)$ is taken as a correctness criterion, then any model, $\text{Exec}_A^0\ l,i$ that matches up to some sequence $SC\ l,i$ in $\{SC\}_i$ terminates the Base Command $\text{Comm}_A^0\ i$.

3(7). In particular (for B a CET heuristic), B may execute $SC\ l,i$ under $\text{Comm}_B^0\ l,i$ to give $\text{Exec}_B^0\ l,i$ and may do so for any value of l (this, for example, is a demonstration).

3(8). In response to a specific $\text{Comm}_A^0\ l,i$ A may execute $\text{Proc}_A^0\ l,i$ if it exists in Π_A^0 .

3(9). More generally A may give $\text{Comm}_A^0\ i$ to (Base Command) either weakest (C^-) interpretation or the strongest (C^+). The C^- interpretation calls for the execution of any one procedure and is thus

$$\text{Proc}_A^0\ l,i \text{ in } \Pi_A^0 \quad (\text{Comm}_A^0\ l,i)$$

3(10). Due to the fact that $D^0(R_i)$ is taken to be an $SC\ l,i$ the strongest (C^+) interpretation is restricted to be parallel and independent execution. That is, though we write

$$\text{Proc}_A^0\ l,i \text{ in } \Pi_A^0 \quad (\text{Comm}_A^0\ l,i)$$

The only legal procedures Proc_A^0 (those appearing in $D^0(R_i)$) constitute serial programs.

3(11). Whichever interpretation A gives to $\text{Comm}_A^0\ i$ (either C^- or C^+) he is (under a CET heuristic), correct, if for any value of l , $\text{Exec}_A^0\ l,i \Leftrightarrow \text{Exec}_B^0\ l,i$.

3(12). The restriction to serial procedures $SC\ l,i$ amounts to excluding the possible $\vee$ or $\bar{\vee}$ disjuncts that arise in $\{SC\}_i$ if there is a common intermediary condition holding. For example, in Diagram II, u is a "common condition holding" as it is the (intermediary) precondition either for $C_2^{\#}$ or $C_3^{\#}$. In principle, for any "common condition" it is possible to introduce a choice or $\vee$ disjunct. Similarly if there is a procedure that computes some condition, $\vee$, such that $C_2^{\#}$ is permitted (given u) if $\vee$ holds and $C_3^{\#}$ is permitted (given u) if $\vee$ does not hold then it is possible to introduce a $\bar{\vee}$ disjunct at U also. Both of these possibilities are excluded (or rendered illegal) by opting to specify $D^0(R_i)$ as $\{SC\}_i$.

3(13). In fact, there is no need to exclude them. If we do not do so, the resulting (more complicated and connected) structure is called a command graph and is symbolized $\{GC\}_i$.

3(14). The l th complete subgraph of any command graph is designated $GC\ l,i$. It may be an $SC\ l,i$ but usually it is a structure involving command form disjuncts.

3(15). If $D^0(R_i)$ is a command graph then A 's weakest or C^- interpretation of $\text{Comm}_A^0\ i$ is

$$\text{Proc}_A^0 l, i \text{ in } \pi_A^0 (\text{Comm}_A^0 l, i)$$

where $\text{Proc}_A^0 l, i$ may legally image any $GC\ l, i$ and A 's strongest or C^+ interpretation of $\text{Comm}_A^0 l, i$ is

$$\text{Proc}_A^0 l, i \text{ in } \pi_A^0 (\text{Comm}_A^0 l, i)$$

which now means concurrent (non-precluded) execution (in contrast to parallel execution).

3(16). Henceforward, the paradigm $D^0(R_i)$ is some $\{GC\}_i$. Several representations of GC_i are discussed between 5(1) and 5(8); at the moment no one representation is superior for all purposes. However, any of these representations can be transformed (usually with some loss of clarity) into an and/or/graph with cyclic substructures adjoined to it (like $D^1(R)$).

3(17). Under a CET heuristic with $D^0(R_i) = GC_i$ A is correct if and only if $\text{Exec}_A^0 l, i \Leftrightarrow \text{Exec}_B^0 l, i$ for some value of l . But here, of course, a command subgraph $GC\ l, i$ is executed.

3(18). The restriction imposed by a one clock modeling facility is that any correct model corresponds to a $GC\ l, i$ which is also an $SC\ l, i$. As a result, certain legitimate models are discounted.

4. Relation Between the $D^0(R_i)$

Returning to the point raised in 2(17), it may or may not be the case that an arbitrary pair of $D^0(R_i)$ have any relation other than their data link connections to $D^1(R)$. For the special case when nodes p, q belong to the same kernel of node i it happens that $D^0(R_i)$, $D^0(R_p)$ and $D^0(R_q)$ refer to the same modeling facility.

In general, this is not the case. For example certain "laws of flow" are common to resistance networks and to water pipes. Knowledge of the same laws could be gleaned from experience in an electrical laboratory or a hydraulic laboratory. These laboratories are related as modeling facilities through $D^1(R)$ but not otherwise. Knowledge that Kirchoff's equations are isomorphic to (some) hydraulic equations is a different matter and depends upon both kinds of experience. The node representing the knowledge is embedded in an analogy substructure of $D^1(R)$ not in a simple disjunction. Notice, for instance that the "real" and "abstract" halves of STATLAB are brought into correspondence, explicitly, by the analogy "probabilistic inference" without which there would be two distinct modeling facilities.

5. Interpretations and Identifications

5(1). Just as the (unadorned) collection $\{SC\}_i$ corresponds, in the abstract, to a set of independent serial programs (which may be represented as flow charts or finite automata) so GC_i , the paradigm $D^0(R_i)$, corresponds, in the abstract, to a Petri Net [Holt 1966, Petri 1961, 23].

5(2). So far as we know, the mathematical theory of Petri Nets is still incomplete. Since his paper, interpreting Petri Nets as computing systems, Holt in concert with Commoner [35] has refined two cases of primary concern to the purist; namely, "occurrence systems" (which admit parallel computation but no interesting interaction) and the converse, nonparallel case, which is an abstracted finite automaton. Though both cases are fully explored, neither is sufficient for the present purpose.

5(3). If the relations expressed in $D^0(R_i)$ are all functions then a combinatonic network [18] is a general representation. But the restriction to functions is severe. It means that the L productions engendered by the execution of $D^0(R_i)$ are verbs in the imperative or the indicative mood; the subjunctive mood is excluded.

5(4). Various steps can be taken to remedy the unwanted restriction to well determined execution.

5(5). One obvious expedient, appeal to the classical theory of finite probabilistic automata, is of no avail. True, the state-transitions of such automata are aleatory; that is, any many-to-one transition rule (or rewriting rule in the corresponding formal language) is applied "probabilistically," in the sense that an outcome is selected by an independent (probabilistically biased) source. But the outcome of a transition is one well determined state (perhaps corresponding to a production in the formal language). To sum it up, the standard sense of "probabilistic" execution is a parody of subjunctive usage achieved by insisting upon a one clocked processor; one and only one of whatever (at stage τ) may become the case must be the case at stage $\tau + 1$.

5(6). Nondeterministic programs, in Manna's sense (our paradigm for a procedure Proc) may or may not have probabilistic assignments or evaluation statements, and are able to express a class of concurrent computations. Zadeh's [86] "Fuzzy Algorithms" have similar (rather broader) capabilities and stem from the quite different substrate of "Fuzzy Set Theory" [84; 85; 74]. Between them, nondeterministic programs and Fuzzy Algorithms comprehend many important concurrent computations and it seems likely that Gaines' [28] independently nurtured unifying theory will bridge these two approaches. In this paper, Manna's formulation is emphasized.

5(7). The restriction highlighted by Manna's formulations is that a nondeterministic program has a beginning and an end; this demarcates an occasion n , and (whatever concurrency or simultaneity or spontaneity may occur within an occasion) all occasions are linearly ordered. The constraint is convenient; for the property "having occasion" is one determinant of a "strict conversation" (in contrast to a "conversation," unqualified). But this property is adopted for convenience, not of necessity. It is certainly not maintained that all conversations are strict.

5(8). Zadeh's [84; 85; 86] (somewhat different) approach makes it especially clear that the result of a fuzzy (or a nondeterministic) computation is neither necessarily nor usually, a state. It is a condition holding; for example, some class of fuzzy states that may, incidentally, form the data for a subsequent computation. This point is important and at the root of the notion "concurrency." At least, this fact justifies our otherwise pedantic insistence upon "condition" and the outright dissavowal of state unless a state, in the strict sense, is intended.

5(9). The C^- interpretation of $\text{Comm}_A^0 i$ involves either choice over a $\cup$ disjunct (our view of choice as truncation of a process) or information to resolve a $\bar{\cup}$ disjunct (obtained, for example, from B ; by way of a cooperative transaction).

5(10). In contrast the C^+ interpretation of $\text{Comm}_A^0 i$ gives rise to the operation of a system which, in McCulloch's [53] sense, has "redundancy of potential command." This activity is simulated by a program such as SRETIC [41] which is executed, in fact, by a one clocked or serial processor. The activity is performed if the system is executed in a many clocked processor.

5(11). In the latter context it is illuminating to note that any synchronizing interaction, due to actualizing a potentially cooperative interaction set out in $D^0(R_i)$, is an overt act of cooperation and information transfer. This may amount to a strict conversation between A and B (the participating P Individuals to whom $\text{Comm}_A^0 i$ and $\text{Comm}_B^0 i$ are addressed) or a conversation involving parts of a participant (A_7 in A ; each one of the A_7 a P Individual). This is the "internal" dialogue referred to the body of the paper. Whereas the transaction between A and B in any strict conversation is truncated by the end of an occasion the internal of dialogue may (or may not) continue as a means of coupling occasions.

5(12). The distinction between the interpretation forms C^+ and C^- is the theoretical backbone for our empirical distinction between holist competence and serialist competence (Pask 1969, 1970, Pask and Scott 1970, 1972) in contrast to the more transparent distinction between a class of holist strategies and serialist strategies.

5(13). If $\text{Comm}_A^0 i$ is addressed to A (similarly, for $\text{Comm}_B^0 i$ addressed to B) then $\text{Comm}_A^0 i$ may be given its C^+ interpretation if α , the L processor for A , is many clocked. If so, the combination, α, A has a holist competence (and A may elect to use a holist strategy). Conversely $\text{Comm}_A^0 i$ may only be given the interpretation C^- if α is one clocked (the same comments apply to B and β). If so, the A, α combination has only a serialist competence.

5(14). As a tenet of the theory of strict conversations any L processor such as α or β must be many clocked; so, at first sight, the class of combinations α, A or B, β with serialist competence, is empty.

The gross account is easily refined to yield a nonempty class and it is interesting to see what was being glossed by the absence of this refinement.

5(15). Recall another tenet of the theory of strict conversations; next rule is in order to recognize an understanding (and thus to recognize the end of an occasion) it is necessary to introduce one distinction of level (L^1, L^0 ; Π_A^1, Π_A^0 ; Π_B^1, Π_B^0 or S_B^1, S_B^0 , $D^1(R), D^0(R)$; "reproducer" and "reproduced").

The distinction of level may be exchanged if desired, for a further "distinction between" or "lack of synchronization of" processor clocks. This possibility is evident from the clock constraint construction of figure 1 and figure 2; conversely, separate clocks are needed to instrument L^1 processes as distinct from L^0 processes and, if the clocks are made explicit, the L^1, L^0 , distinction can be discarded.

5(16). Thus, if a conversation is viewed at one level as it is when scrutinizing a learning strategy or a performance strategy then a two-clocked processor (legitimate as α or β) appears as though it is a one-clocked system, backed up by a reproductive

equipment clocked otherwise. Under these circumstances certain permissible A , α and B , β combinations are endowed with serialist competence only; they act as though they are one-clocked processor if viewed at one level only.

5(17). Whatever has been said of Base Commands like $\text{Comm}_A^0 i = \langle A! R_i / \text{Precon } i \rangle$ applies equally to the interrogative form $\text{EQuest}_A i = \langle A? \text{Expl } i / R_i \rangle$.

Similarly, all comments about qualified commands ($\text{Comm}_A j i$) apply to qualified questions ($\text{EQuest}_A j i$).

5(18). Under interrogation, an explanation $\text{Expl}_A^0 i$ of R_i is a path or a class of paths, marking out condition holdings.

5(19). Due to the uniform representation prescribed in 3(10) the construction rules for $D^1(R)$ and $D(R_i)$ are structurally identical, being given different interpretations, at level L^1 and level L^0 . For example the “no interference” requirement imposed upon any $D^1(R)$ corresponds to the “no conflict” requirement imposed upon $D^0(R_i)$. Similarly the forms of Comm_A^0 , Comm_A^1 ; Exec_A^0 , Exec_A^1 ; EQuest_A^0 , EQuest_A^1 ; Expl_A^0 , Expl_A^1 ; are isomorphic.

5(20). Hence, if the appropriate interpretation is given (for an external observer's convenience) it is legitimate to write (with “ $\rightarrow$ ” standing for “gives rise to”) a learning strategy under the CET heuristic (over occasions $n = 0, 1, \dots, n, n + 1, \dots, N$) as

$$\begin{array}{ll}
 \text{Comm}_A^1 i_0 \longrightarrow \text{Exec}_A^1 i_0 & \text{EQuest}_A^1 i_0 \longrightarrow \text{Expl}_A^1 i_0 \Leftarrow D^1(R_{i_0}) \\
 \text{Comm}_A^1 i_1 \longrightarrow \text{Exec}_A^1 i_1 & \text{EQuest}_A^1 i_1 \longrightarrow \text{Expl}_A^1 i_1 \Leftarrow D^1(R_{i_1}) \\
 \text{or as} & \vdots \\
 \text{Comm}_A^1 i_n \longrightarrow \text{Exec}_A^1 i_n & \text{EQuest}_A^1 i_n \longrightarrow \text{Expl}_A^1 i_n \Leftarrow D^1(R_{i_n}) \\
 \text{Comm}_A^1 i_{n+1} \longrightarrow \text{Exec}_A^1 i_{n+1} & \text{EQuest}_A^1 i_{n+1} \longrightarrow \text{Expl}_A^1 i_{n+1} \Leftarrow D^1(R_{i_{n+1}}) \\
 \text{Comm}_A^1 i_N \longrightarrow \text{Exec}_A^1 i_N & \text{EQuest}_A^1 i_N \longrightarrow \text{Expl}_A^1 i_N \Leftarrow D^1(R_{i_N})
 \end{array}$$

To set $\text{Comm}_A^0 i_0 = \langle A! R_{\text{Head}} / \Pi^0(0) \rangle$ (and $\text{EQuest}_A^0 i_0$, similarly); to “exit,” if the command/question is terminated/answered; if not; to demand that $i_N = \text{Head}$ in $D^1(R)$ and for each n to set and require both

$$\text{Comm}_A^0 i_n \longrightarrow \text{Exec}_A^0 i_n \Leftrightarrow \text{Exec}_B^0 i_n \Leftarrow D^0(R_{i_n})$$

and

$$\text{EQuest}_A^0 i_n \longrightarrow \text{Exec}_A^0 i_n \Leftrightarrow \text{Expl}_B^0 i_n \Leftarrow D^0(R_{i_n})$$

as “ A understanding R_{i_n} ” where, for each model $\{x\}_{t_n}$ in $\text{Exec}_A^0 i_n$, t_n belongs to occasion n which is ended by an understanding of R_{i_n} . In particular occasion n ends (for

exit) with A understanding R_{Head} . So that $\langle A! R_{\text{in}}/\Pi^0(N) \rangle = \langle A! R_{\text{Head}}/\Pi^0(N) \rangle$ is terminated.

5(21). The L^1 statement of 5(20) is a learning strategy. The contextual coverage of Section 2(12) applies in the context of this statement (taken up to the n th occasion).

5(22). Just as $D^0(R_j)$ can be taken prescriptively so may $D^1(R)$. Any L^1 prescription by B to A is a teaching strategy.

Appendix C

1. Consider the least specific meaning an observer might attach to an M Individual, i.e. the minimal M Individual. It is a spatially localized physical or biological apparatus, a brain or a computing machine of any kind whatsoever, that does one or more (unspecified) operation(s) when one or more (unspecified) condition(s) are satisfied together with the following requirement; the apparatus does (unconditionally) the operations needed to make it persist (those operations needed to reproduce it; to render it stable). Thus the Minimal M Individual is a physically embodied conditional imperative. To give it a less high sounding, but more illuminating, title; it is a processor specified without reference to the indefinitely large number of procedures it might execute. If presented with a procedure that nominates the missing operation(s) and condition(s) the processor "gives an imperative interpretation to" i.e. it "executes" the procedure.

2. All minimal specifications are parodies of reality and, in common with many others, this particular specification is a little bizzare. Most of the M Individuals we choose to distinguish have specific and inbuilt operations over and above those required to maintain the processor's integrity. For example, human and animal brains are bound by innate constraints, to operations that appear (on description) as characteristic behaviors. Only some of the brain (most of the human brain, very little of the amphibian brain) is free to act as a processor in the sense of a program space to be inhabited by procedures.¹³

3. The constraints are more easily explained in terms of electronic mechanism rather than biology. In these terms, a processor comprises an assembly of clocks, of shift registers and of stores.

From a traditional stance, where integer indexing, is taken for granted (and, with it, the ideas of one locus control of one state at once and one clock of) the simplest paradigm is probably a material embodiment (in real fabric) of a universal Turing Machine. The material embodiment of an abstract digital computer, a "programmed machine" [55] or a "synchronous, centrally regulated, programmable, iterative equipment with a Boolean Algebra over its states set" [17] is more convenient and realistic since in the latter case, several clocks may operate if they are strictly synchronized.

4. For technical reasons we cannot restrict our attention to this "simple" class of processor though it is usefully cited as a category for which a great deal of background knowledge is available regarding what happens if an initial state (conditions and

¹³"Most" and "little" are used properly in this context as demarcating relative sizes, numerosities of neurones, connecting fibers and so on. Hence, they carry no immediate commitment regarding relative information capacity. If the processor class is restricted, they may do. For example, if the processor is of the type later reserved for environments then capacity bounds can be established through Bremmerman's limit [13; 1-5].

operations for the conditional imperative) is specified by a program and if the system is left to run. In addition, it is at least necessary to countenance processors that are able to execute procedures permitting parallel and concurrent modes of computation. For example, combinatonic networks [18], combinatory tessellation automat (Chiarviglio, Barralt, Torrijos, and DeMilo 1972) some abstract graph automata [72; 34; 68] are procedures of this type which call for less constrained processors.

5. Technical considerations apart, it is desirable to make the class of processor as general as possible. The word "simple" in the last paragraph, was intentionally placed between inverted commas. The processor class under discussion at that point is deemed "simple" for chiefly historical reasons that stem from two developments. It happens that computer design (of physical objects, *M* Individuals, like "abstract digital computers") was initiated by the mathematical discoveries of Markoff, Church, Post, and others; in turn, the success of these devices encouraged mathematicians to work on similar lines. The interaction was especially obtrusive inside the embryonic computer industry of the early 1950s. Because these ways of describing and designing machines as string processors became familiar, they are deemed "simple" as from that point of view, they should be. But there is nothing in electronics to make them so and if hybrid computer designs had won out at that date, as they are doing so today, then the notion of simplicity would be very different. By the same token, but with much greater cogency, it would be unwise to dwell on apparently "simple" processors if the brain is taken as an *M* Individual (as it mostly is, in this paper), or, for that matter, any other large physical system such as a polymerizing aggregate, a star, or a cyclic biochemical reaction.

Moreover, generality pays dividends in a different direction. During a later contribution to this series of papers, we find it useful to regard various types of vehicle, spatio temporally attached to the driver's/pilot's brain, as *M* Individuals in their own right; thus, to extend the idea of processing to physically productive operations such as moving around in space subject to restrictions upon velocity, acceleration, and the like. For most vehicle/brain *M* Individuals the processor constraints are simple enough, but they do not have the same forms as those bearing upon a "simple" string processor.

6. The type of *M* Individual constraint depends upon the specificity with which the processor, common to all such entities, is isolated by an observer. He may, for example, count organizations like compilers and overall executive routines as processors constraints if these normally programmatic objects are immutably bound to the *M* Individual. Generally, we mean, by *M* Individual, whatever constraints the computer scientist refers to as hardware or storage organization.

7. With psychological processes in mind, an external observer is likely to *M* Individuate entities like human brains or, in view of recent work on other organisms, some animal brains as well; for example, Dolphin brains ([46; 47] or Chimpanzee brains if they are augmented, by the apparatus needed to establish meaningful communication (????). But a modicum of caution should be exercised for reasons of the following kind.

Any functional human brain reacts; for instance any effective stimulus gives rise to an electrical response in the frontal cortex that images a usually larger response in the appropriate sensory region. But, under these circumstances, there is no evidence that the brain is acting as an *L*-Processor and there is ample circumstantial evidence that it is not

doing so. Apparently the brain acts as an *L*-Processor if and only if it is organized by an orientating process, involving the cerebral cortex, the reticular formation, and the limbic system into a special state, manifest in average EEG records as the "Contingent negative Variation" (Gray Walter 1968). Given this organization, the brain, qua *M* Individual, is an *L* processor. It may execute any *P* Individual which momentarily, at least, inhabits the brain in question. Moreover, the *P* Individual is one case of the original orientation. If executed, this *P* Individual entertains an *L*-hypothesis and anticipates an outcome; loosely speaking "the brain does so." Under these conditions, as Gray Walter points out, an external observer can legitimately speak of expectation, information, redundancy, equivocation, relevance, and so on with respect to the hypothesis in hand; otherwise, these terms are rightfully used in respect of the observer's constructs only (just as an observer might speak of "information transfer," computed from the entries in a contingency table representing arbitrary experimental data).

8. It would be out of place to pursue the assumptions underlying all this. The example is used to teach one simple lesson; namely that *M* Individuals like brains, which are "obviously" candidate *L*-Processors do not invariably act in this capacity. On the other side of the coin, *M* Individuals that are often neglected because they are inanimate (the computing machines and polymerizing aggregates mentioned earlier) may often serve as perfectly good *L* Processors.

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